

# RESEARCH MEMORANDUM

ANALYSIS OF LIMITATIONS IMPOSED ON ONE-SPOOL TURBOJET -  
ENGINE DESIGNS BY TURBINES HAVING DOWNSTREAM STATORS

AT 0, 2.0, AND 2.8 FLIGHT MACH NUMBERS

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SUMMARY

An aerodynamic design-point analysis was made of one-spool turbojet engines having one-stage turbines with one and with two rows of downstream stator blades. The objects of this analysis were to evaluate the design characteristics of such turbines in comparison with conventional one- and two-stage turbines, to determine the extent to which exit whirl can be increased before causing weight-flow capacity to decrease, and to determine the effect of downstream staters on engine design limitations. Relative weights of the various types of turbine are not considered in the comparisons made.

The results of this analysis show that the capacities of one-stage turbines with downstream staters exceed those of conventional one-stage turbines but fall far short of conventional conservative two-stage turbines. The improvement in capacity of one-stage turbines with downstream staters over that of conventional high-output one-stage turbines is sufficiently large to warrant the use of the former type of turbine design in preference even to conventional conservative two-stage designs for applications requiring turbine cooling, since this task is simpler if only one rotor must be cooled. One-stage turbines with two downstream staters of 0.2 diffusion factor compare favorably with and in some instances surpass the capacities of similar turbines having one or two downstream staters of 0.4 or 0.6 diffusion factor. Compared with conventional one-stage turbines, turbines with downstream staters do not require such a high compressor rotor-inlet relative Mach number at the high flight Mach number condition; so that greater latitude in compressor equivalent overspeed capacity should be available for use in off-design operation. A value of about -0.6 for turbine rotor-exit tangential Mach number (exit whirl) yields maximum turbine weight-flow capacity for a given obtainable compressor pressure ratio and turbine stress. The addition of downstream staters alleviates the turbine stress problem to a small extent.

## INTRODUCTION

In order to investigate the possibilities of turbines with downstream stators, an aerodynamic analysis was made of such turbines in reference 1, and the results were presented in the form of design charts. The function of the downstream stators is to turn the flow at the rotor exit to the axial direction. This action permits the use of high design rotor-exit whirl without the excessively high exhaust-nozzle losses usually associated with this velocity component in conventional turbines (turbines without downstream stators). Because of the higher exit whirl, higher specific work can be expected from a one-stage turbine with downstream stators than from a conventional one-stage turbine having the same aerodynamic limits. In the present discussion, all one-stage turbines, conventional or otherwise, are high-output designs. (In this report, high-output turbines are those limited to 0.8 relative Mach number at the rotor inlet; conservative turbines are those limited to 0.6 relative Mach number.)

An important item that cannot be discerned from the charts of reference 1 in their present form is the effect of increasing turbine-exit whirl on equivalent weight flow per unit turbine frontal area (hereafter referred to as turbine-limited specific weight flow, or simply weight-flow capacity). If the turbine rotor-exit axial Mach number is maintained constant, as in reference 1, an increase in turbine-exit whirl is accompanied by a decrease in turbine-exit specific weight flow parameter. The feasibility of turbines with downstream stators depends in part upon whether or not the increase in specific work due to high exit whirl is great enough to offset the decrease in turbine-exit specific weight flow parameter to yield sufficiently higher turbine-limited specific weight flow than a conventional one-stage turbine.

The method of analysis used in reference 2 proved convenient in translating the turbine data from the charts and tables of reference 3 into engine parameters. In reference 2, engine design performance characteristics in terms of obtainable compressor pressure ratio and turbine-limited specific weight flow of conventional one- and two-stage turbines are presented on a single map. Such maps reveal a noticeable gap between the weight-flow and work capacities of high-output one-stage turbines and those of conservative two-stage turbines. The analysis in reference 1 was made with the expectation that one-stage turbines with downstream stators might fill this gap.

For this report, charts similar to those of reference 2, but for turbines with downstream stators, have been constructed from the turbine design data of reference 1. This aerodynamic analysis of one-spool turbojet engines was made at the NACA Lewis laboratory. The objects of the present report are (1) to compare these design charts for one-stage turbines having downstream stators with those for conventional one- and

two-stage turbines, (2) to compare the charts of various designs of turbines with downstream stators and thus indicate the extent to which exit whirl can be increased before weight-flow capacity decreases, and (3) to evaluate the effect of the addition of a downstream stator on engine design limitations. The present analysis is entirely aerodynamic; in the comparisons that are made, relative weights of the various types of turbine are not considered.

In order to prevent high velocity from adversely affecting the performance of the engine components downstream of the turbine, diffusion of the exhaust gases is generally provided for. The use of high exit whirl means that the absolute velocity at the turbine exit will be high. Therefore, a large amount of diffusion will be required of a downstream stator in addition to a large amount of turning of the flow. In compressor theory, a blade-loading parameter called the diffusion factor is used that effectively relates the amount of diffusion and the associated loss. The diffusion factor is an index of the amount of diffusion of the flow that a blade row accomplishes. Currently, a diffusion factor of 0.2 is considered conservative, 0.4 is moderate, and 0.6 is critical. For a given value of diffusion factor, two rows of downstream stator blades should provide both greater diffusion and more turning of the flow than one row of downstream stator blades.

The turbine design charts of this report have been prepared for engine temperature ratios of 3.0 and 4.0 and flight conditions that range from a flight Mach number of 0 at sea level to 2.8 in the stratosphere. All turbine designs are limited at the hub radius by 0.8 rotor-inlet relative Mach number, 0.7 rotor-exit axial Mach number, and a maximum  $120^\circ$  turning of the relative flow direction through the rotor. Both one and two downstream stators are considered, a straight annulus being assumed for the downstream stators in every instance. Diffusion factors of 0.2, 0.4, and 0.6 are assigned for designs having one downstream stator. For designs having two downstream stators, diffusion factors of 0.2 and 0.4 are assumed.

## ANALYSIS AND CONSTRUCTION OF CHARTS

### Analysis

Just as in reference 2, the present analysis is formulated around the so-called parameter  $e$ ,  $wU_t^2/A_t\delta_1'\sqrt{\theta_1}$ , which is convenient for relating compressors and turbines. The symbols used are defined in appendix A. Parameter  $e$  is simply the product of equivalent weight flow per unit frontal area and the square of the equivalent blade tip speed. Parameters  $e$  for compressors and turbines are related by

$$\frac{w_T U_{t,T}^2}{A_T \delta_1' \sqrt{\theta_1'}} = (1 + f)(1 - b) \frac{w_C U_{t,C}^2}{A_C \delta_1' \sqrt{\theta_1'}} \quad (1)$$

For specified values of fuel-air ratio  $f$  and compressor bleed  $b$  (including any leakage), parameters  $e$  for compressors and turbines are related by a constant factor. If the fuel-air ratio and compressor bleed are small, little error is made in assuming that parameters  $e$  for compressors and turbines are equal.

Parameter  $e$  has the following significance. Use of a high value of parameter  $e$  for a specified compressor pressure ratio and engine temperature ratio results in a high value of turbine-limited specific weight flow for turbines of given aerodynamic limits. This high turbine weight-flow capacity is obtained, however, at the expense of increased compressor-inlet relative Mach number and increased turbine blade centrifugal stress. Therefore, in the discussion that follows, a decrease in the value of parameter  $e$  is indicative of a decrease in the severity of compressor design; for given weight-flow and work capacities, therefore, low values of parameter  $e$  are sought. Current laboratory design techniques permit designing for compressor-inlet relative Mach numbers of 1.2 with the expectation of good compressor performance. Reference 2 shows that, at this 1.2 compressor-inlet relative Mach number, the greatest value of  $e$  is 44 million lb/sec<sup>3</sup> for a compressor hub-tip radius ratio of 0.4.

The present analysis is concerned with the manipulation of the turbine design parameters of the charts in reference 1 into forms convenient for producing design-characteristic turbine charts similar to the type presented in reference 2. Because the data of reference 1 are employed in the present analysis, the assumptions used in that reference again apply. These assumptions are reviewed in appendix B, along with a tabulation of the constants used in the present analysis. Figure 1 is a sketch showing the location of the numerical stations in the engine.

In the production of the charts of the present report (figs. 1 to 5), turbine-limited specific weight flow  $w_T \sqrt{\theta_1'} / A_T \delta_1'$ , compressor pressure ratio  $p_2'/p_1'$ , turbine equivalent blade tip speed  $U_{t,T} / \sqrt{\theta_1'}$ , and parameter  $e$  were determined by applying the following equations to the readings of the charts of reference 1.

From the continuity relation, the turbine-limited specific weight flow is

$$\frac{w_T \sqrt{\theta_1'}}{A_T \delta_1'} = \frac{2116}{\sqrt{518.7}} \sqrt{\frac{2kg}{(k+1)R}} \left( \frac{\rho V_x}{\rho' a_{cr}'} \right)_{5,m} \left[ 1 - \left( \frac{r_h}{r_t} \right)^2 \right] \frac{p_5'}{p_1'} \frac{1}{\sqrt{\frac{T_5'}{T_1'}}} \quad (2)$$

In order that  $w_T \sqrt{\theta_1} / A_T \delta_1$  can be calculated from equation (2), determinations of  $p_5'/p_1'$  and  $T_5'/T_1'$  are necessary. The ratio  $T_5'/T_1'$  is expressible as

$$\frac{T_5'}{T_1'} = \frac{T_3'}{T_1'} \frac{T_5'}{T_3'} \quad (3)$$

and  $p_5'/p_1'$  as

$$\frac{p_5'}{p_1'} = \frac{p_2'}{p_1'} \frac{p_3'}{p_2'} \frac{p_5'}{p_3'} \quad (4)$$

Equation (14) of reference 1 expresses  $T_5'/T_3'$  in terms of the dimensionless parameters used to present the data in this reference as

$$\frac{T_5'}{T_3'} = 1 - 2 \frac{k-1}{k+1} \left( \frac{-gJ\Delta h'}{U_{t,T}^2} \right) \left( \frac{U_{t,T}}{a'_{cr,3}} \right)^2 \quad (5)$$

where the negative sign indicates a drop in enthalpy. The compressor pressure ratio can be determined from the stage-work parameter and blade-speed parameter by

$$\frac{p_2'}{p_1'} = \left[ 1 + \frac{2k}{k+1} \frac{\gamma-1}{\gamma} \eta_C \frac{T_3'}{T_1'} \left( \frac{-gJ\Delta h'}{U_{t,T}^2} \right) \left( \frac{U_{t,T}}{a'_{cr,3}} \right)^2 \right]^{\frac{\gamma}{\gamma-1}} \quad (6)$$

This parameter, as well as being the abscissa of the charts presented herein, is a necessary item for evaluating  $w_T \sqrt{\theta_1} / A_T \delta_1$  (eqs. (2) and (4)). Turbine pressure ratio, which is also needed in equations (2) and (4), is calculated from

$$\frac{p_5'}{p_3'} = \left[ 1 - 2 \frac{k-1}{k+1} \frac{1}{\eta_T} \left( \frac{-gJ\Delta h'}{U_{t,T}^2} \right) \left( \frac{U_{t,T}}{a'_{cr,3}} \right)^2 \right]^{\frac{k}{k-1}} \quad (7)$$

Turbine equivalent blade tip speed can be expressed as

$$\frac{U_{t,T}}{\sqrt{\theta_1'}} = \sqrt{\frac{2k}{k+1}} 518.7 \text{ gR} \sqrt{\frac{T_3'}{T_1'}} \frac{U_{t,T}}{a'_{cr,3}} \quad (8)$$

In reference 2, equation (E9) for parameter  $e$  is derived:

$$\frac{w_T U_{t,T}^2}{A_T \delta_1 \sqrt{\theta_1}} = \frac{2116}{\sqrt{518.7}} 288 \sqrt{\frac{2kg^3}{(k+1)R}} \left( \frac{\rho V_x}{\rho' a'_{cr}} \right)_{5,m} \frac{S}{\theta_1' \Gamma_T w_T} \times$$

$$\frac{\left[ 1 - \frac{\gamma}{\gamma-1} \frac{k-1}{k} \frac{\left( \frac{p_2'}{p_1'} \right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_T \eta_C \frac{T_3'}{T_1'}} \right]^{\frac{k}{k-1}} \frac{p_2'}{p_1'} \frac{p_3'}{p_2'}}{\left[ 1 - \frac{\gamma}{\gamma-1} \frac{k-1}{k} \frac{\left( \frac{p_2'}{p_1'} \right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_C \frac{T_3'}{T_1'}} \right]^{1/2} \sqrt{\frac{T_3'}{T_1'}}} \quad (9)$$

where the turbine-exit specific weight flow parameter is given by

$$\left( \frac{\rho V_x}{\rho' a'_{cr}} \right)_{5,m} = \left[ 1 - \frac{k-1}{k+1} \left( \frac{V_x}{a'_{cr}} \right)_{5,m}^2 - \frac{k-1}{k+1} \left( \frac{V_u}{a'_{cr}} \right)_{5,m}^2 \right]^{\frac{1}{k-1}} \left( \frac{V_x}{a'_{cr}} \right)_{5,m} \quad (10)$$

The turbine-exit specific weight flow parameter varies with turbine hub-tip radius ratio, diffusion factor, and the number of rows of downstream stator blades considered; its determination was one of the necessary steps in making the downstream stator analysis of reference 1. By combining equations (2) to (8), equation (9) was solved for turbine blade centrifugal stress as the dependent variable.

Equation (9) shows that, for a given turbine blade centrifugal stress  $S$ , parameter  $e$  is a function of  $\theta_1'$ , compressor pressure ratio  $p_2'/p_1'$ , engine temperature ratio  $T_3'/T_1'$ , and turbine-exit specific weight flow parameter  $(\rho V_x/\rho' a'_{cr})_{5,m}$ , if compressor and turbine adiabatic efficiencies are assumed constant. But for the variable  $\theta_1'$ , which is a function of flight Mach number and altitude, the entire analysis presented herein could be presented independent of flight conditions. The analysis, however, can be generalized to apply to all flight conditions if equivalent turbine blade centrifugal stress  $S/\theta_1'$  be considered as an independent variable in equation (9).

In order to use the preceding equations in preparing the turbine charts, values were assigned to certain aerodynamic parameters; these values are summarized in table I. Furthermore, ranges of values were assigned to the following two turbine aerodynamic parameters for use in entering the charts of reference 1:  $r_h/r_t$  and  $U_{t,T}/a'_{cr,3}$ . Table I also serves as an index of the flight conditions investigated herein.

### Construction of Charts

Turning angle. - In all the charts presented, the turbine rotor annular area is assumed straight as long as the relative turning angle of the flow at the hub radius of the turbine rotor is less than  $120^\circ$ . When the turning angle reaches  $120^\circ$ , the turbine rotor annular area is assumed to diverge in such a manner that the relative turning angle remains constant at  $120^\circ$ . Reference 1 shows that the result of suppressing the relative turning angle by as much as  $23^\circ$ , to maintain the  $120^\circ$  limit, is to forfeit only 5 percent of the work.

Plotting procedure. - From the data obtained from the analysis of reference 1, the following plotting procedure was used to produce the turbine charts.

Design-characteristic charts: Preliminary plots were constructed of turbine-limited specific weight flow  $w_T \sqrt{\theta'_1/A_T \delta'_1}$  against compressor pressure ratio  $p'_2/p'_1$  for lines of constant turbine-inlet blade-speed parameter  $U_{t,T}/a'_{cr,3}$ , each such plot being drawn for a given diffusion factor, engine temperature ratio, and number of downstream stators. Similarly, preliminary plots of parameter  $e$  against turbine-limited specific weight flow  $w_T \sqrt{\theta'_1/A_T \delta'_1}$  were made. Preliminary plots were also made of turbine blade centrifugal stress  $S$  against compressor pressure ratio  $p'_2/p'_1$  for lines of constant turbine hub-tip radius ratio  $r_h/r_t$ , each plot being drawn for one of the assigned values of diffusion factor, engine temperature ratio, number of downstream stators, and flight Mach number. The design-characteristic charts presented are cross plots of these preliminary plots. Figure 2(a) shows design characteristics for sea-level static designs and Mach 1.28 designs in the stratosphere; figures 3(a) and 4(a), for Mach 2.0 designs in the stratosphere; and figures 5(a) to (e), for Mach 2.8 designs in the stratosphere.

Table II of reference 2 lists for various engine temperature ratios the compressor pressure ratios yielding minimum specific fuel consumption for afterburning engines. Arrows designate these particular compressor pressure ratios on the design-characteristic turbine charts presented herein.



Cross plots: In order to show most vividly the relative merits of one-stage turbines having downstream stators in comparison with conventional one- and two-stage turbines, various supplementary cross plots have been made of portions of the design-characteristic turbine charts.

In figure 2(b), turbine-limited specific weight flow and parameter  $e$  are plotted against compressor pressure ratio for one-stage turbines with one row of downstream stator blades of 0.4 diffusion factor and for conventional one- and two-stage turbines, for a constant value of turbine blade centrifugal stress of 30,000 pounds per square inch. This figure applies to sea-level static designs and Mach 1.28 designs in the stratosphere, the values of turbine-limited specific weight flow  $w_T \sqrt{\theta_1' / A_T \delta_1'}$  and parameter  $e$  for the plots having been read from figure 2(a) and from figure 4(a) in reference 2. Figures 2(c) and (d) parallel figure 2(b) for turbine blade centrifugal stress values of 40,000 and 50,000 pounds per square inch, respectively. The dotted lines on all the supplementary cross plots for parameter  $e$  indicate the current limit of 44 million lb/sec<sup>3</sup>.

Similarly, figures 3(b) and (c) are presented for Mach 2.0 flight in the stratosphere at an engine temperature ratio of 3.0, and figures 4(b) and (c) apply to Mach 2.0 flight in the stratosphere at an engine temperature ratio of 4.0. These figures for Mach 2.0 flight are presented for turbine blade centrifugal stresses of 30,000 and 50,000 pounds per square inch. No curves are shown in figure 3(c) for two-stage turbines, because in the range of interest of compressor pressure ratio, turbine hub-tip radius ratios would be lower than that for which data are readily available in reference 3. Data for figures 3(b) and (c) and 4(b) and (c) were read directly from figures 3(a) and 4(a) for turbines with downstream stators, and from figures 4(b) and (e) of reference 2 for the conventional one- and two-stage turbines.

Figures 5(f) and (g) are similar cross plots showing the variations of turbine-limited specific weight flow and parameter  $e$  with compressor pressure ratio for Mach 2.8 operation at an engine temperature ratio of 3.0. The additional curves shown on these figures appear because of the several combinations of diffusion factor and number of downstream stators investigated for this particular flight condition. The data for figures 5(f) and (g) were read from figures 5(a) to (e).

Figures 5(h) and (i) are cross plots illustrating the effect on turbine-limited specific weight flow of increasing exit whirl from the turbine rotor that is brought about by increasing diffusion factor and number of rows of downstream turbine stator blades. These curves are shown for three values of compressor pressure ratio, as well as for turbine stresses of 30,000 and 50,000 pounds per square inch. These figures likewise show the variation of compressor pressure ratio with exit whirl for constant turbine-limited specific weight flow. The isolated

circles locating the corresponding points for conventional one-stage turbines were read from figure 4(h) of reference 2, and are therefore analytical. Figures 5(a) to (e) furnished the rest of the data for figures 5(h) and (i), except for the points at the highest value of the abscissa, which were determined by means of data in reference 1.

## RESULTS AND DISCUSSION

In this discussion, values of compressor pressure ratio, turbine-limited specific weight flow, turbine blade centrifugal stress, and parameter  $e$  are used as bases for comparisons. Values of these parameters read from the charts developed herein are compared with those of the conventional one- and two-stage turbines of reference 2. Furthermore, these same parameters serve as the basis of ascertaining engine design limitations.

### Sea-Level Static Designs and Mach 1.28 Designs in Stratosphere

Turbine-inlet temperature of 2075° R. - The following tables contain design variables read from figure 2(a) for one-stage turbines with one downstream stator of 0.4 diffusion factor. The engine temperature ratio is 4.0. For the purpose of comparison, values of the same parameters for one- and two-stage turbines, read from reference 2 are also tabulated.

Turbine design	$p_2'/p_1'$	$\frac{w_T \sqrt{\theta_1'}}{A_T \delta_1'}$ , lb/(sec)(sq ft)	Percent increase in work capacity over conventional one-stage turbine
Conventional one-stage	6.20	25.0	0
One-stage with one downstream stator	7.55	27.3	14

The preceding table shows that, at the current limit of 30,000 pounds per square inch on turbine blade centrifugal stress and of 44 million lb/sec<sup>3</sup> on parameter  $e$ , a one-stage turbine with one row of downstream stator blades can produce 14 percent more work while having a 9 percent greater weight-flow capacity than a conventional one-stage turbine with the same aerodynamic limits. Figure 2(b) shows that, for the conditions of this table, a one-stage turbine with one downstream stator lies just about intermediate between a conventional one-stage and a conventional two-stage turbine.

The following table illustrates the relative merits of these turbines on the basis of maintaining substantially the same compressor pressure ratio and turbine-limited specific weight flow for all three types:

Turbine design	S, psi	$p_2'/p_1'$	$\frac{w_T \sqrt{\theta_1'}}{A_T \delta_1'}$ , lb/(sec)(sq ft)	Parameter e, lb/sec <sup>3</sup>
Conventional one-stage	50,000	7.25	32	$77 \times 10^6$
One-stage with one downstream stator	40,000	7.20	32	58
Conventional conserva- tive two-stage	30,000	7.25	33	37

If, then, by turbine cooling, the limit on turbine blade centrifugal stress could be raised to 50,000 pounds per square inch, a conventional one-stage turbine could produce a compressor pressure ratio of 7.25 and a turbine-limited specific weight flow of 32 pounds per second per square foot. Parameter e is so high for this design, 77 million lb/sec<sup>3</sup>, that the current range of efficient compressor design is exceeded. A one-stage turbine with a downstream stator can, for substantially the same weight-flow and work capacity, moderate both the problems of turbine stress and compressor aerodynamics. The turbine blade centrifugal stress is reduced to 40,000 pounds per square inch and parameter e to 58 million lb/sec<sup>3</sup> (which can be achieved with a compressor-inlet relative Mach number of 1.33). An orthodox conservative two-stage turbine, however, can completely solve both problems.

Sea-level static designs require high compressor pressure ratio and high turbine-limited specific weight flow. According to reference 2, a compressor pressure ratio of 11.6 for afterburning engines yields minimum specific fuel consumption for an engine temperature ratio of 4.0. This reference further discusses the desirability of designing for compressor pressure ratios either at or below this particular value. For this reason, the following table is presented on the basis of a value of 11.5 for compressor pressure ratio:

Turbine design	S, psi	$\frac{w_T \sqrt{\theta_1'}}{A_T \delta_1'}$ , lb/(sec)(sq ft)
Conventional one-stage	26,000	19.2
One-stage with one downstream stator	28,000	22.0
Conventional conserva- tive two-stage	33,000	32.5

At the limit of 44 million lb/sec<sup>3</sup> on parameter  $e$ , a one-stage turbine with one downstream stator has 15 percent greater weight-flow capacity than an orthodox one-stage turbine at the 11.5 compressor pressure ratio. At this value of compressor pressure ratio and of parameter  $e$ , a conventional conservative two-stage turbine has an additional 55 percent greater capacity of turbine-limited specific weight flow. Figures 2(b) and (c) show that at the high compressor pressure ratios the values of turbine-limited specific weight flow of the one-stage turbine with a downstream stator become increasingly lower than those of the orthodox conservative two-stage turbine. These same figures show that the curves of parameter  $e$  for the turbine with a downstream stator maintain their relative positions between those of one- and two-stage turbines, with increasing compressor pressure ratio.

Critical comment. - Designed for sea-level static operation (or Mach 1.28 in the stratosphere), a one-stage turbine with a downstream stator of 0.4 diffusion factor has a 14 percent greater work capacity and a 9 percent greater turbine-limited specific weight flow than an orthodox one-stage turbine at a stress of 30,000 pounds per square inch and a parameter  $e$  of 44 million lb/sec<sup>3</sup>. While an orthodox conservative two-stage turbine can completely solve the problems of both turbine stress and compressor aerodynamics, a one-stage turbine with a downstream stator can mitigate these problems. At the high weight-flow and work capacities required at sea-level static designs, which are unobtainable from conventional one-stage turbines within present turbine stress and compressor aerodynamic limits, a one-stage turbine with one downstream stator is adequate but is still less capable than an orthodox conservative two-stage turbine. For a given turbine weight-flow and work capacity, a one-stage turbine with one downstream stator moderates both the problems of turbine stress and compressor aerodynamics in comparison with a conventional one-stage turbine. For engine designs having a one-stage turbine with a downstream stator, compressor aerodynamics becomes limiting at a turbine blade centrifugal stress of 28,000 pounds per square inch. The corresponding stresses for engine designs having conventional one- and two-stage turbines are 26,000 and 33,000 pounds per square inch, respectively.

#### Mach 2.0 Designs in Stratosphere

Turbine-inlet temperature of 2106° R. - At an engine temperature ratio of 3.0, for which figure 3 is constructed, a compressor pressure ratio of 5.5 yields minimum specific fuel consumption for an afterburning engine (ref. 2). Thus, compressor pressure ratio is preferably restricted to values of 5.5 or less.

The following table lists values of parameters read from figure 3(a), along with values taken from reference 2 (fig. 4(b)) for comparison:

Turbine design	$\frac{w_T \sqrt{\theta_1'}}{A_T \delta_1'},$ lb/(sec)(sq ft)	Parameter $e,$ lb/sec <sup>3</sup>
Conventional one-stage	18.5	$26.5 \times 10^6$
One-stage with one downstream stator	20.8	24.6
Conventional conservative two-stage	25.0	21.0
Conventional high-output two-stage	34.0	26.5

At a centrifugal stress of 30,000 pounds per square inch in the turbine rotor blades and a compressor pressure ratio of 5.0, a one-stage turbine with one row of downstream stator blades exceeds a conventional one-stage turbine by 12 percent in weight-flow capacity, with a 7-percent reduction in parameter  $e$ . A conservative two-stage turbine, nevertheless, has an additional 23 percent greater weight-flow capacity. A conventional high-output two-stage turbine is far superior even to the conservative two-stage turbine. Because a turbine-inlet temperature of  $2106^\circ \text{R}$  might be attainable without turbine cooling, an orthodox conservative two-stage turbine should prove most feasible for this particular application.

Figure 3(b) shows that, with respect to turbine-limited specific weight flow at low compressor pressure ratio, the performance of one-stage turbines with downstream stators lies intermediate between that of conventional one- and two-stage turbines. At higher compressor pressure ratios, the performances approach those of the conventional one-stage turbines.

The next table presents a comparison at a turbine blade centrifugal stress of 50,000 pounds per square inch and a compressor pressure ratio of 5.0:

Turbine design	$\frac{w_T \sqrt{\theta_1'}}{A_T \delta_1'},$ lb/(sec)(sq ft)	Parameter $e,$ lb/sec <sup>3</sup>
Conventional one-stage	24.3	$44.0 \times 10^6$
One-stage with one downstream stator	26.6	41.5

Under these conditions, and with parameter  $e$  at approximately its current limit for efficient compressor operation, a one-stage turbine with one row of downstream stator blades has a 9 percent greater weight-flow capacity than a conventional one-stage turbine. Figure 3(c) shows that this increase is maintained over the range of compressor pressure ratios of interest. Figure 3(c) further shows the constant margin of lower values of parameter  $e$  that is obtainable from one-stage turbines with a downstream stator as compressor pressure ratio is varied.

Turbine-inlet temperature of 2808° R. - Figure 4(a) is identical with figure 2(a), except for the stress lines. The following table lists values of parameters read from figure 4(a) along with values taken from reference 2 for a stress of 30,000 pounds per square inch. A compressor pressure ratio of 10 was selected as a basis of comparison, this value being slightly lower than that (11.6) for minimum specific fuel consumption of afterburning engines, for this particular engine temperature ratio of 4.0:

Turbine design	$\frac{w_T \sqrt{\theta_1}}{A_T \delta_1},$ lb/(sec)(sq ft)	Parameter $e,$ lb/sec <sup>3</sup>
Conventional one-stage	18.0	$36.5 \times 10^6$
One-stage with one downstream stator	20.2	34.0
Conventional conserva- tive two-stage	28.0	29.0

This table shows that either at the current limits on turbine blade centrifugal stress or on parameter  $e$ , turbines with one row of downstream stator blades more nearly approximate the capacities of conventional high-output one-stage turbines than of orthodox conservative two-stage turbines. This trend is vividly shown at low and high stress levels by figures 4(b) and (c), being more pronounced as compressor pressure ratio increases. For the same compressor pressure ratio of 10, a one-stage turbine with one downstream stator has 12 percent greater weight-flow capacity at the stress limit of 30,000 pounds per square inch than a conventional high-output one-stage turbine.

Although a conventional conservative two-stage turbine exceeds a one-stage turbine with a downstream stator in weight-flow capacity by 39 percent, the latter design might prove the more practical for this particular application. The high turbine-inlet temperature of 2808° R will undoubtedly necessitate turbine cooling. Since cooling a stator is much simpler than cooling a rotor, the over-all task of cooling the two stators and the rotor of a one-stage turbine with a downstream

stator should be preferable to that of cooling the two stators and two rotors of a conventional two-stage turbine. Furthermore, the 12-percent increase in weight-flow capacity over a conventional one-stage turbine might justify the small added complexity of cooling the downstream stator.

The following table presents a comparison on the basis of a compressor pressure ratio of 10 at a turbine blade centrifugal stress of 50,000 pounds per square inch:

Turbine design	$\frac{w_T \sqrt{\theta_1}}{A_T \delta_1'}$ , lb/(sec)(sq ft)	Parameter e, lb/sec <sup>3</sup>
Conventional one-stage	25.5	60.5×10 <sup>6</sup>
One-stage with one downstream stator	27.9	57.6
Conventional conserva- tive two-stage	35.2	48.5

A one-stage turbine with a downstream stator not only has 9 percent greater turbine-limited specific weight flow, but also its value of parameter e is less by 5 percent.

Critical comment. - For Mach 2.0 flight in the stratosphere at an engine temperature ratio of 3.0, a conventional conservative two-stage turbine appears to be most practical. For the same obtainable compressor pressure ratio and a turbine blade centrifugal stress of 30,000 pounds per square inch, a conservative two-stage turbine exceeds a one-stage turbine with one downstream stator by 20 percent in weight-flow capacity. The 2106° R turbine-inlet temperature should require no turbine cooling.

For flight at a Mach number of 2.0 in the stratosphere and at an engine temperature ratio of 4.0, a one-stage turbine with one downstream stator appears to be preferable to a conventional conservative two-stage turbine, despite the 39 percent greater weight-flow capacity of the latter for the same obtainable compressor pressure ratio and turbine blade centrifugal stress. The reason for this conclusion is that the 2808° R turbine-inlet temperature will require turbine cooling, which is much more complicated for the two stators and two rotors of the two-stage turbine than for the two stators and one rotor of the one-stage turbine with one downstream stator.

Whereas, at the present limit on parameter e and at a compressor pressure ratio of 11.5, a one-stage turbine with one downstream stator of 0.4 diffusion factor exceeds a conventional one-stage turbine by 15 percent in weight-flow capacity for sea-level static operation, the difference decreases to 12 percent for Mach 2.0 flight in the stratosphere, for

the same engine temperature ratio of 4.0 and turbine stress of 30,000 pounds per square inch. For Mach 2.0 flight in the stratosphere, the improvement in weight-flow capacity of one-stage turbines with one downstream stator over that of a conventional one-stage turbine is greater at 30,000 than at 50,000 pounds per square inch. For a specified turbine stress and work output, a one-stage turbine with a downstream stator not only yields improvement in weight-flow capacity over a conventional one-stage turbine but also causes a drop in parameter  $e$ , the lowering of which relieves the severity of compressor aerodynamics.

For Mach 2.0 engine designs in the stratosphere, compressor aerodynamics does not become a limiting factor until the turbine blade centrifugal stress reaches values from 39,000 to 52,000 pounds per square inch, depending on the turbine-inlet temperature. The corresponding values for conventional one-stage turbines are 37,000 to 49,000; hence the use of the downstream stator can relieve the compressor aerodynamic problem to a small degree. Turbine blade centrifugal stress commences to impose limitations at this flight condition, as is the case for conventional one- and two-stage turbines.

#### Mach 2.8 Designs in Stratosphere

Turbine-inlet temperature of 3004° R. - All the turbine charts for Mach 2.8 designs at a turbine-inlet temperature of 3004° R were constructed for an engine temperature ratio of 3.0. These charts (fig. 5) are presented for one-stage turbines having one and two downstream stators with various values of diffusion factor. Comparison of the capacities of these turbines among themselves as well as with the conventional one- and two-stage turbines of reference 2 is made with several tabulations of parameters read from these charts, as well as from the supplementary cross plots.

In the following table, a compressor pressure ratio of 5.0 and a stress of 30,000 pounds per square inch are used as a basis of comparison (as for Mach 2.0 designs having an engine temperature ratio of 3.0):



Turbine design	$D_h$	$\frac{w_T \sqrt{\theta_1}}{A_T \delta_1}$ lb/(sec)(sq ft)	Parameter $e$ , lb/sec <sup>3</sup>
Conventional one-stage		14.5	$18.5 \times 10^6$
One-stage with one downstream stator	0.2	16.0	$18.3 \times 10^6$
	.4	16.8	17.1
	.6	17.2	16.0
One-stage with two downstream stators	0.2	16.6	$17.1 \times 10^6$
	.4	16.9	15.2
Conventional conservative two-stage		21.2	$14.6 \times 10^6$
Conventional high output two-stage		29.5	$18.5 \times 10^6$

The table shows that, whereas a conventional high-output one-stage turbine has a weight-flow capacity of only 14.5 pounds per second per square foot at these conditions, a one-stage turbine having one downstream stator with a conservative diffusion factor of 0.2 can produce 16 pounds per second per square foot, an 11-percent increase. As the diffusion factor is increased from 0.2 to 0.4 to 0.6, the weight-flow capacity increases an additional 6 and 3 percent, respectively. The turbine-limited specific weight flows of similar one-stage turbines with two downstream stators having diffusion factors of 0.2 and 0.4 are only slightly less than that for a one-stage turbine having one downstream stator with a critical diffusion factor of 0.6. Because a one-stage turbine with two downstream stators of 0.4 diffusion factor yields an 18 percent lower value of parameter  $e$  than a conventional one-stage turbine, the former design permits greater overspeed capacity to be built into the compressor. A conventional conservative two-stage turbine has a weight-flow capacity of 21.2 pounds per second per square foot, which is an additional increase of 30 percent over that of a one-stage turbine having two downstream stators of 0.4 diffusion factor. Furthermore, a conventional high-output two-stage turbine has a weight-flow capacity of 29.5 pounds per second per square foot, which is more than double that of a conventional high-output one-stage turbine.

Figure 5(f) illustrates the marked superiority with respect to weight-flow capacity of two-stage turbines over one-stage turbines with downstream stators. With the exception of a turbine with one downstream

stator of 0.2 diffusion factor, there is little to choose from among the other types of design with respect to turbine-limited specific weight flow. With respect to parameter  $e$ , however, a one-stage turbine with two downstream stators of 0.4 diffusion factor has nearly as low a value of parameter  $e$  over the range of compressor pressure ratios presented as does a conventional conservative two-stage turbine.

The next table presents readings of figure 5 at a compressor pressure ratio of 3.0 and a stress of 30,000 pounds per square inch:

Turbine design	$D_h$	$\frac{w_T \sqrt{\theta_1}}{A_T \delta_1^2}$ , lb/(sec)(sq ft)	Parameter $e$ , lb/sec <sup>3</sup>
Conventional one-stage		18.2	$16.5 \times 10^6$
One-stage with one downstream stator	0.2	20.7	$16.1 \times 10^6$
	.4	22.1	15.2
	.6	22.6	14.2
One-stage with two downstream stators	0.2	22.2	$15.5 \times 10^6$
	.4	22.3	13.5

In this tabulation, the same trend of weight-flow variation with diffusion factor is observed as for a compressor pressure ratio of 5.0 and stress of 30,000 pounds per square inch. Again, the value of parameter  $e$  can be reduced by as much as 18 percent by the use of one-stage turbines with two downstream stators. Values are not included for conventional two-stage turbines, because the blade speed is very low at this compressor pressure ratio. Here, again, a turbine with one downstream stator of 0.6 diffusion factor yields highest turbine-limited specific weight flow.

The next table illustrates the effect of varying the number of rows of downstream stator blades and of diffusion factor on obtainable compressor pressure ratio, in which the basis of comparison is constant values of 30,000 pounds per square inch for turbine stress and 16.0 pounds per second per square foot for turbine-limited specific weight flow:

Turbine design	$D_h$	$p_2'/p_1'$	Parameter $e$ , lb/sec <sup>3</sup>	Percent increase in work capacity over conventional one-stage turbine
Conventional one-stage		4.05	$17.8 \times 10^6$	0
One-stage with one downstream stator	0.2	5.00	$18.4 \times 10^6$	19
	.4	5.37	17.3	25
	.6	5.53	16.0	28
One-stage with two downstream stators	0.2	5.29	$17.1 \times 10^6$	24
	.4	5.42	15.3	26

Use of one downstream stator of 0.2 diffusion factor with a one-stage turbine results in a 19-percent increase in turbine specific work. If the diffusion factors of one downstream stator are increased to 0.4 and 0.6, turbine specific work continues to increase. The obtainable work output of a one-stage turbine having two rows of downstream stator blades of 0.2 diffusion factor is but 1 percent less than that of a one-stage turbine with one downstream stator of 0.4 diffusion factor; furthermore, it is even less than 2 percent lower than a similar turbine with two downstream stators of 0.4 diffusion factor. This observation, which is also illustrated by figure 5(f), is of interest; because, if experiment should reveal that a diffusion factor of 0.4 (which is considered moderate in compressor design) is too critical for use in turbine design, a diffusion factor of 0.2, which is more likely to result in low loss in stagnation pressure across the downstream stators, can be designed into two rows of stator blades and will yield comparable capacity. The value of parameter  $e$  for the one-stage turbine with two downstream stators of 0.2 diffusion factor is substantially the same as that for the one-stage turbine with one downstream stator of 0.4 diffusion factor.

The following tabulation is presented to compare the capacities of the various turbine types at high turbine blade centrifugal stress (50,000 psi):

Turbine design	$D_h$	$\frac{w_T \sqrt{\theta_1}}{A_T \delta_1},$ lb/(sec)(sq ft)	Parameter $e,$ lb/sec <sup>3</sup>
Conventional one-stage		20.2	$31.0 \times 10^6$
One-stage with one downstream stator	0.2	21.6	$30.5 \times 10^6$
	.4	22.6	29.0
	.6	22.5	26.7
One-stage with two downstream stators	0.2	22.5	$29.0 \times 10^6$
	.4	22.1	25.5
Conventional conservative two-stage		27.0	$24.7 \times 10^6$
Conventional high-output two-stage		36.0	$31.0 \times 10^6$

At a stress of 50,000 pounds per square inch and a pressure ratio of 5.0, a one-stage turbine with one downstream stator of 0.4 diffusion factor yields the highest weight-flow capacity of the turbines with downstream stators. Figure 5(g) shows that this observation holds true over the range of compressor pressure ratios of interest. For a turbine having one downstream stator of 0.6 diffusion factor, the turbine-limited specific weight flow decreases from its value for 0.4 diffusion factor, because the turbine-exit specific weight flow parameter  $(\rho V_x / \rho' a'_{cr})_{5,m}$  decreases for constant  $(V_x/a)_{5,h}$  as exit whirl  $-(V_u/a)_{5,h}$  is increased. This decrease, in this particular instance, more than offsets the increase in weight-flow capacity due to added work capacity. A turbine with two downstream stators of 0.2 diffusion factor has a weight-flow capacity nearly equal to that of a turbine with one downstream stator of 0.4 diffusion factor. Furthermore, its value of parameter  $e$  is identical. Although the use of downstream stators can produce a 12-percent increase in weight-flow capacity, this still does not approach the 27.0 pounds per second per square foot that can be obtained from a conventional conservative two-stage turbine.

Figures 5(h) and (i) show clearly how the weight-flow capacity of a turbine increases with increase in turbine rotor-exit whirl to a maximum and then drops off in value, for constant compressor pressure ratio. The curves of compressor pressure ratio for selected constant values of turbine-limited specific weight flow do likewise. The reason for the dropping off of turbine-limited specific weight flow or of compressor pressure ratio has already been explained herein in terms of the decrease in turbine rotor-exit specific weight flow parameter  $(\rho V_x / \rho' a'_{cr})_{5,m}$  with increase in exit whirl as  $(V_x/a)_{5,h}$  is maintained constant.

At a stress of 30,000 pounds per square inch (fig. 5(h)), the curves for turbine-limited specific weight flow peak between values of -0.6 to -0.7 for  $(V_w/a)_{5,h}$ ; the peaks for compressor pressure ratio occur between values of -0.5 to -0.7 for  $(V_w/a)_{5,h}$ . It cannot be said, however, that the best design would be that at a compromise of these two peaks, even though such a compromise might yield the most desirable turbine design. Examination of the magnitude of parameter  $e$  to the right of these peaks is illuminating. For example, the peaks of the curves in figures 5(h) and (i) correspond to magnitudes of exit whirl less than that used with a one-stage turbine with two downstream stators of 0.4 diffusion factor. Yet figures 5(f) and (g) show that this very type of design yields the lowest value of parameter  $e$ , a characteristic that can provide for greater equivalent overspeed capacity of the compressor.

A summary of turbine design characteristics is presented in table II to illustrate at a glance their variation with flight Mach number and turbine stress. Values shown in table II differ in some instances from those in the short tables previously presented, because in table II only compressor pressure ratios close to those for minimum specific fuel consumption and only stress values of 30,000 and 50,000 pounds per square inch are listed. Thus, values of parameter  $e$  occasionally become excessive. Table II, therefore, makes a comparison of turbine designs solely with regard to turbine aerodynamics and neglects consideration of compressor aerodynamics. Table II shows that for constant turbine blade centrifugal stress the improvement in weight-flow capacity of one-stage turbines having downstream stators over that of conventional one-stage turbines increases with increase in flight Mach number. For the same flight Mach number, the improvement in weight-flow capacity decreases with increasing turbine stress.

Critical comment. - Designed for a flight Mach number of 2.8 in the stratosphere and an engine temperature ratio of 3.0, a one-stage turbine with a downstream stator has up to 24 percent more weight-flow capacity than a conventional one-stage turbine for the same turbine aerodynamic limits, obtainable compressor pressure ratio, and turbine stress (see p. 17). Or alternatively, for the same turbine aerodynamic limits, turbine-limited specific weight flow, and turbine stress, the increase in work output over a conventional one-stage turbine is as much as 28 percent (see p. 18). Although a conventional conservative two-stage turbine has 23 percent greater weight-flow capacity (see p. 16) than the best of the one-stage turbines with downstream stators, the latter type of design might prove more feasible because it is simpler to cool.

Increase of diffusion factor, which permits increase of turbine rotor-exit whirl, does not necessarily yield continually increasing weight-flow capacity for a given value of compressor pressure ratio. Rather, the weight-flow capacity begins to decrease at values of exit whirl of the order of -0.6.

In the event that experiment should disclose that a value of 0.4 for diffusion factor is too high to yield efficient performance in turbines, one-stage turbines with downstream stators should nevertheless remain in consideration. A one-stage turbine with two rows of downstream stator blades having a conservative diffusion factor of 0.2 rivals and in some instances exceeds the capacities of turbines with one or two downstream stators of 0.4 and 0.6 diffusion factors.

Whereas, for Mach 2.0 flight in the stratosphere a one-stage turbine with one downstream stator of 0.4 diffusion factor exceeds a conventional one-stage turbine by 12 percent in weight-flow capacity, the difference increases to 16 percent for Mach 2.8 flight in the stratosphere, for the same engine temperature ratio of 3.0, turbine blade centrifugal stress of 30,000 pounds per square inch, and compressor pressure ratio of 5.0. The improvement obtainable from one-stage turbines with one downstream stator is greater at a centrifugal stress of 30,000 than at 50,000 pounds per square inch. Improvements in weight-flow and work capacities of one-stage turbines with downstream stators over those of conventional one-stage turbines are accompanied by a condition (lower parameter  $e$ ) permitting greater overspeed capacity of the compressor.

As was found in reference 2 for conventional one- and two-stage turbines, compressor aerodynamics imposes no limitation for engine designs of 2.8 flight Mach number when one-stage turbines with one or two downstream stators are used. Turbine blade centrifugal stress is likewise the primary constraint for this flight condition. For one-stage turbines with one or two rows of downstream stators, high weight-flow capacities are obtainable only at turbine stress levels of the order of 50,000 to 60,000 pounds per square inch.

### CONCLUSIONS

The following are the main conclusions drawn from the aerodynamic design-point analysis presented:

1. The weight-flow capacities of one-stage turbines with downstream stators exceed those of conventional high-output one-stage turbines having the same aerodynamic limits, obtainable compressor pressure ratio, and turbine stress. This improvement becomes greatest at a flight Mach number of 2.8, a turbine blade centrifugal stress of 30,000 pounds per square inch, and low values of compressor pressure ratio.

2. Conventional conservative two-stage turbines far surpass in capacity one-stage turbines with downstream stators at all three flight conditions investigated and are, therefore, more feasible for designs requiring no turbine cooling. If turbine-inlet temperature is sufficiently high to necessitate turbine cooling, one-stage turbines with

downstream stators might prove most practical; because, whereas their capacities only moderately exceed those of conventional one-stage turbines, their cooling problem is simpler than that of two-stage turbines.

3. One-stage turbines with two rows of downstream stator blades of 0.2 diffusion factor rival and sometimes exceed in capacity similar turbines with one or two downstream stators having diffusion factors of 0.4 and 0.6.

4. For the high-speed flight operation in which a high degree of compressor equivalent overspeed capacity is desirable, a one-stage turbine with downstream stators provides a greater degree of this desirable characteristic than a conventional one-stage turbine.

5. With respect to maximum weight-flow capacity for a given obtainable compressor pressure ratio and turbine stress, a magnitude of about -0.6 for turbine rotor-exit tangential Mach number (exit whirl) is optimum.

6. For one-stage turbines with downstream stators, compressor aerodynamics imposes a severe limitation on static sea-level engine designs. Compressor aerodynamics does not become a limiting factor for Mach 2.0 designs in the stratosphere until turbine stress becomes 39,000 to 52,000 pounds per square inch. For Mach 2.8 designs in the stratosphere, compressor aerodynamics is not limiting.

7. For design-point operation of turbojet engines having turbines with downstream stators, turbine blade centrifugal stress becomes a limiting factor at 2.0 flight Mach number in the stratosphere and is a primary constraint at 2.8 flight Mach number.

Lewis Flight Propulsion Laboratory  
National Advisory Committee for Aeronautics  
Cleveland, Ohio, October 21, 1954

## APPENDIX A

## SYMBOLS

The following symbols are used in this report:

A	annular area, sq ft
A <sub>C</sub>	compressor frontal area, sq ft
A <sub>T</sub>	turbine frontal area, sq ft
a	sonic velocity, $\sqrt{kgRT}$ , ft/sec
a' <sub>cr</sub>	critical velocity relative to stator, $\sqrt{\frac{2k}{k+1} gRT}$ , ft/sec
D <sub>h</sub>	diffusion factor, $1 - \left(\frac{V_6}{V_5}\right)_h + \left(\frac{V_{u,6} - V_{u,5}}{2\sigma V_5}\right)_h$
e	parameter used in relating compressors and turbines, $w_u^2/A_t \delta_1' \sqrt{\theta_1'}$ , lb/sec <sup>3</sup>
g	acceleration due to gravity, 32.17 ft/sec <sup>2</sup>
h	specific enthalpy, Btu/lb
J	mechanical equivalent of heat, 778.2 ft-lb/Btu
k	ratio of specific heats for hot gas, 4/3
M	Mach number relative to rotating blades
p	absolute pressure, lb/sq ft
R	gas constant, 53.4 ft-lb/(lb)(°R)
r	radius, ft
S	turbine blade centrifugal stress at hub radius, psi
sfc	specific fuel consumption, lb fuel/(lb thrust)(hr)
T	absolute temperature, °R
U	blade velocity, ft/sec
V	absolute velocity of gas, ft/sec



W	relative velocity of gas, ft/sec
w	weight flow of gas, lb/sec
$\beta$	flow angle of relative velocity measured from tangential direction, deg
$\Gamma$	density of turbine blade metal, lb/cu ft
$\gamma$	ratio of specific heats for air, 1.4
$\delta$	ratio of pressure to NACA standard sea-level pressure, $p/2116.22$
$\eta$	adiabatic efficiency
$\theta$	ratio of temperature to NACA standard sea-level temperature, $T/518.688$
$\rho$	density of gas, lb/cu ft
$\sigma$	solidity, ratio of aerodynamic chord to pitch
$\psi$	stress-correction factor for tapered blades
$\bar{\omega}$	stagnation-pressure loss coefficient of downstream stator blades

## Subscripts:

C	compressor
h	hub
m	mean
T	turbine
t	tip
u	tangential component
x	axial component
0	free stream
1	compressor inlet
2	compressor exit

- 3 turbine inlet
- 4 turbine rotor inlet
- 5 turbine rotor exit
- 6 exit from first downstream stator
- 7 exit from second downstream stator

Superscript:

- ' stagnation state relative to stator

## APPENDIX B

## ASSUMPTIONS AND CONSTANTS

## Assumptions

The following assumptions were used in making this analysis:

- (1) Simplified radial equilibrium
- (2) Free-vortex velocity distribution
- (3) At the mean radius,  $(\rho V_x)_m A$  equal to integrated value of weight flow over blade height:

$$w = (\rho V_x)_{1,m} A_1 = (\rho V_x)_{2,m} A_2$$

- (4) No radial variation in stagnation state relative to stator
- (5) Hub and mean radii constant in value from inlet to outlet of rotor
- (6) Constant annular area across downstream stators
- (7) Allowable amount of exit whirl determined by ability of downstream stators to turn flow back to axial direction

## Constants

The constants used in the analysis are as follows:

Compressor adiabatic efficiency, $\eta_C$ . . . . .	0.85
Combustor stagnation pressure ratio, $p_3'/p_2'$ . . . . .	0.95
Density of turbine blade metal, $\Gamma$ , lb/cu ft . . . . .	500
Maximum turbine rotor hub turning angle, $\Delta\beta_h$ , deg . . . . .	120
Solidity of downstream stators at mean radius, $\sigma_m$ . . . . .	1.5
Stress-correction factor for tapered blades, $\psi$ . . . . .	0.7
Turbine rotor adiabatic efficiency, $\eta_T$ . . . . .	0.85
Turbine rotor-exit axial Mach number, $(V_x/a)_{5,h}$ . . . . .	0.7
Turbine rotor-inlet relative Mach number, $(W/a)_{4,h}$ . . . . .	0.8

## REFERENCES

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2. Cavicchi, Richard H., and English, Robert E.: Analysis of Limitations Imposed on One-Spool Turbojet-Engine Designs by Compressors and Turbines at Flight Mach Numbers of 0, 2.0, and 2.8. NACA RM E54F21a, 1954.
3. Cavicchi, Richard H., and English, Robert E.: A Rapid Method for Use in Design of Turbines within Specified Aerodynamic Limits. NACA TN 2905, 1953.

TABLE I. - AERODYNAMIC PARAMETERS PRESENTED IN TURBINE CHARTS

[Turbine rotor-inlet relative Mach number,  
 $(W/a)_{4,h}$ , 0.8 for all charts.]

Altitude	Rows of downstream stators	$M_0$	$T_3',$ $^{\circ}R$	$\frac{T_3'}{T_1'}$	$D_h$	$\bar{\omega}$	Figure
Sea level	1	0	2075	4	0.4	0.05	2
Strato- sphere	1	1.28	2075	4	0.4	0.05	2
		2.0	2106	3	0.4	0.05	3
			2808	4	.4	.05	4
		2.8	3004	3	0.2	0.05	5(a)
					.4	.05	5(b)
					.6	.07	5(c)
	2	2.8	3004	3	0.2	0.05	5(d)
					.4	.05	5(e)

TABLE II. - TURBINE DESIGN PARAMETERS OBTAINABLE AT VARIOUS FLIGHT MACH NUMBERS

Altitude	$M_0$	$T_3',$ $^{\circ}R$	$\frac{T_3'}{T_1'}$	S, psi	Turbine design <sup>1</sup>	$D_h$	$\frac{P_2'}{P_1'}$	$\frac{w_T \sqrt{\theta_1}}{A_1 \delta_1'}$ $\frac{lb}{(sec)(sq ft)}$	Parameter $e,$ $lb/sec^3$	Percent in- crease in weight-flow capacity over one-stage turbine
Sea level	0	2075	4	30,000	a	---	11.5	21.5	$50.8 \times 10^6$	0
					b	0.4	↓	23.0	47.6	7
					c	---	↓	31.2	40.5	45
				50,000	a	---	11.5	29.5	$84.6 \times 10^6$	0
					b	0.4	↓	30.8	79.1	4
Strato- sphere	2.0	2106	3	30,000	a	---	5.5	17.5	$26.7 \times 10^6$	0
					b	0.4	↓	19.6	24.9	12
					c	---	↓	24.5	21.2	40
					d	---	↓	33.6	26.7	92
				50,000	a	---	5.5	23.5	$44.5 \times 10^6$	0
					b	0.4	↓	25.6	42.0	9
		2808	4	30,000	a	---	10.0	18.0	$36.5 \times 10^6$	0
					b	0.4	↓	20.2	34.2	12
					c	---	↓	27.6	29.0	53
				50,000	a	---	11.5	24.3	$62.1 \times 10^6$	0
					b	0.4	↓	26.3	58.5	8
					c	---	↓	34.4	49.4	42
	2.8	3004	3	30,000	a	---	5.5	13.8	$18.7 \times 10^6$	0
					b	0.2	↓	15.0	18.6	9
					b	.4	↓	15.8	17.4	14
					b	.6	↓	16.1	16.0	17
					e	.2	↓	15.5	17.1	12
					e	.4	↓	15.8	15.3	14
					c	---	↓	20.4	14.9	48
					d	---	↓	28.8	18.7	109
				50,000	a	---	5.5	19.3	$31.3 \times 10^6$	0
					b	0.2	↓	20.6	30.8	7
					b	.4	↓	21.5	29.3	11
					b	.6	↓	21.4	27.1	11
					e	.2	↓	21.4	29.3	11
					e	.4	↓	21.0	25.8	9
					c	---	↓	26.2	24.9	36

<sup>1</sup>Turbine designs:

- a. Conventional one-stage.
- b. One-stage with one downstream stator.
- c. Conventional conservative two-stage.
- d. Conventional high-output two-stage.
- e. One-stage with two downstream stators.

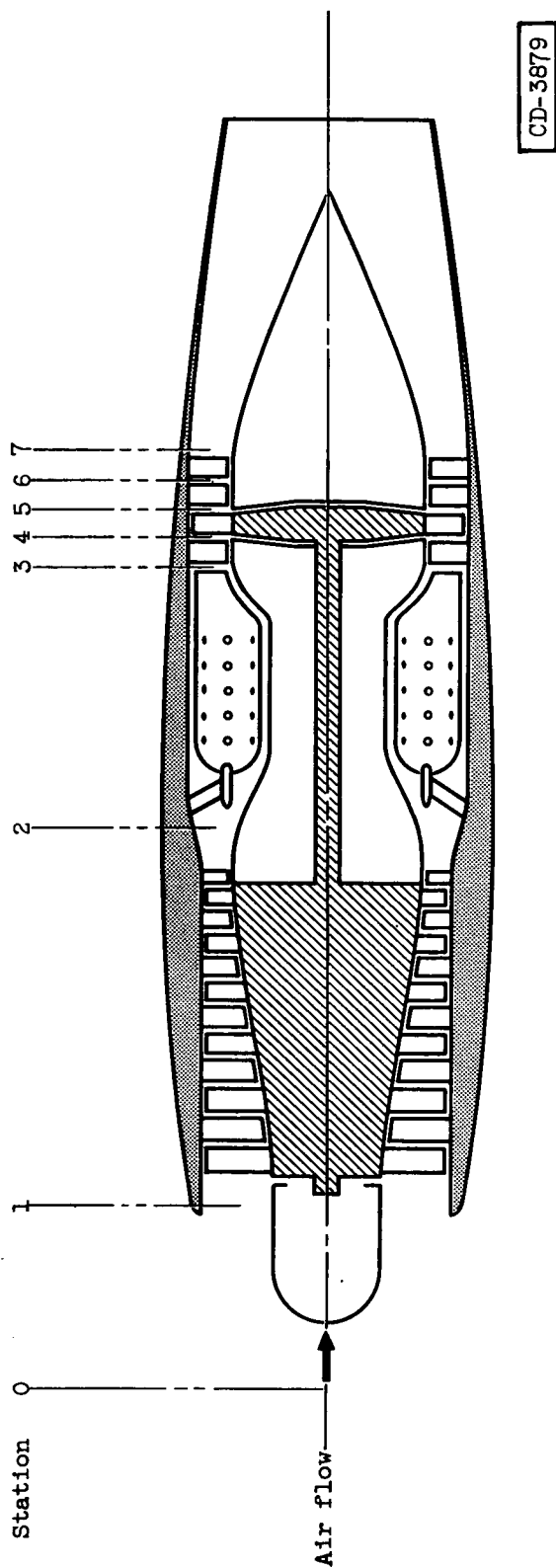
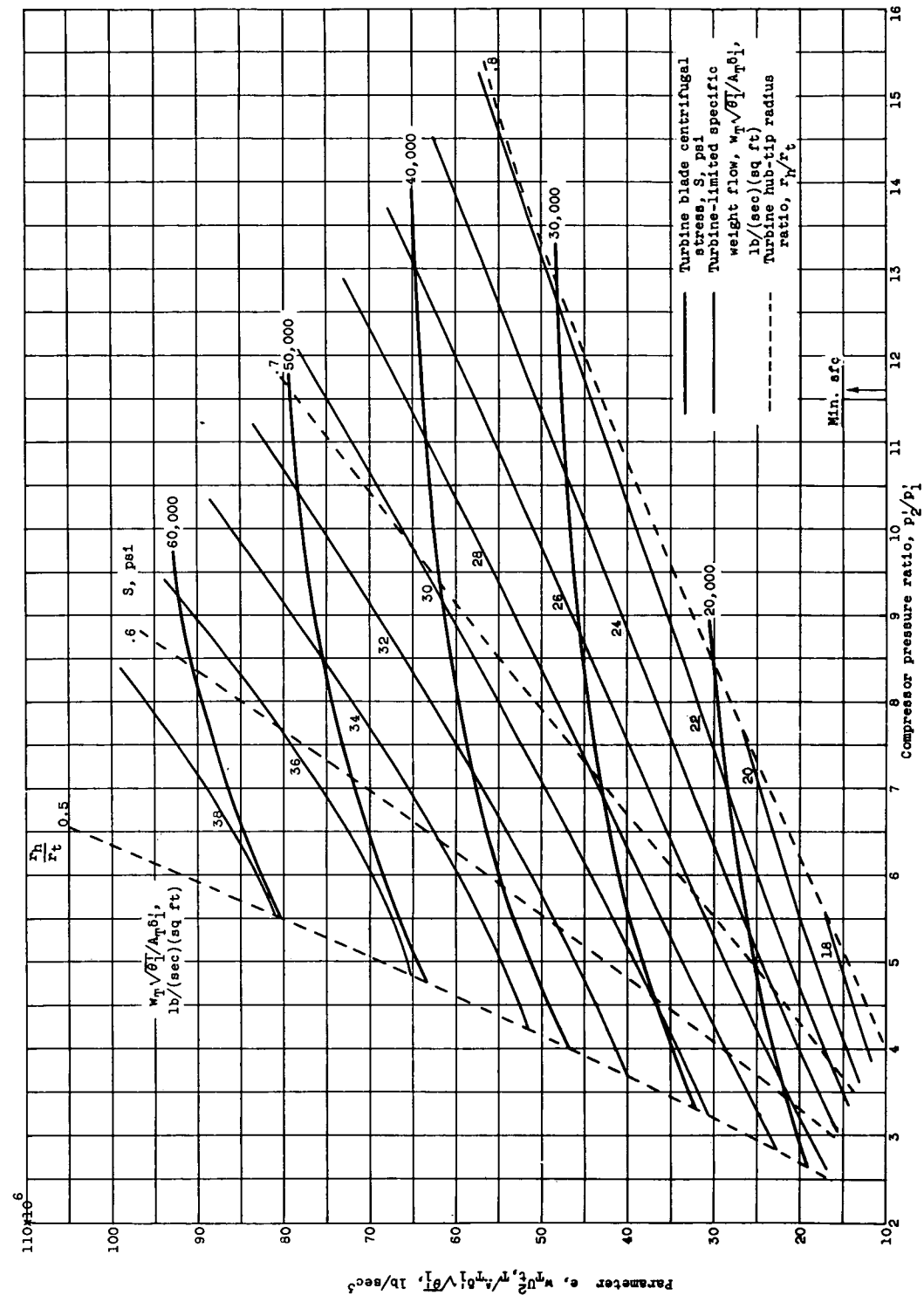
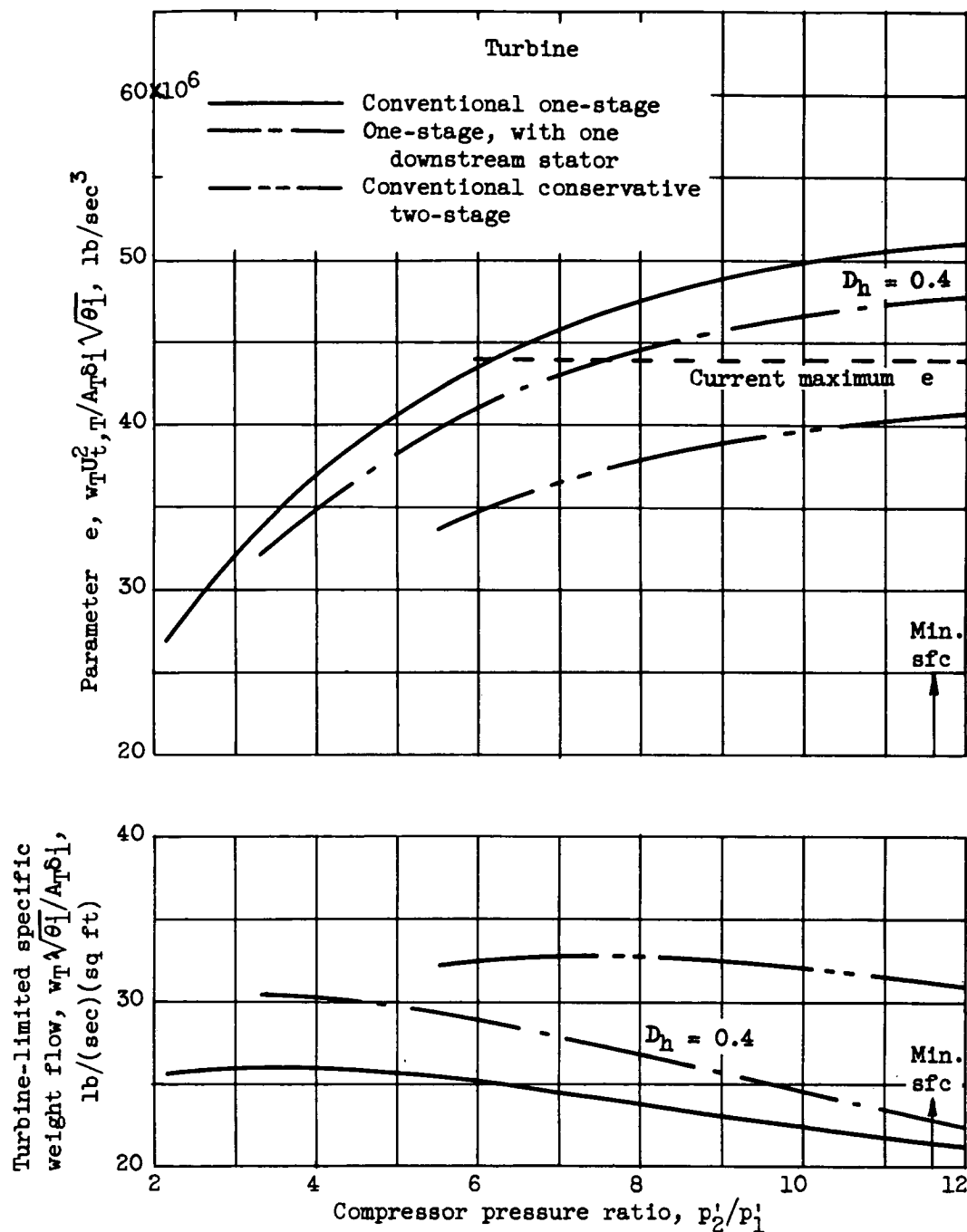


Figure 1. - Cross section of turbojet engine showing location of numerical stations.

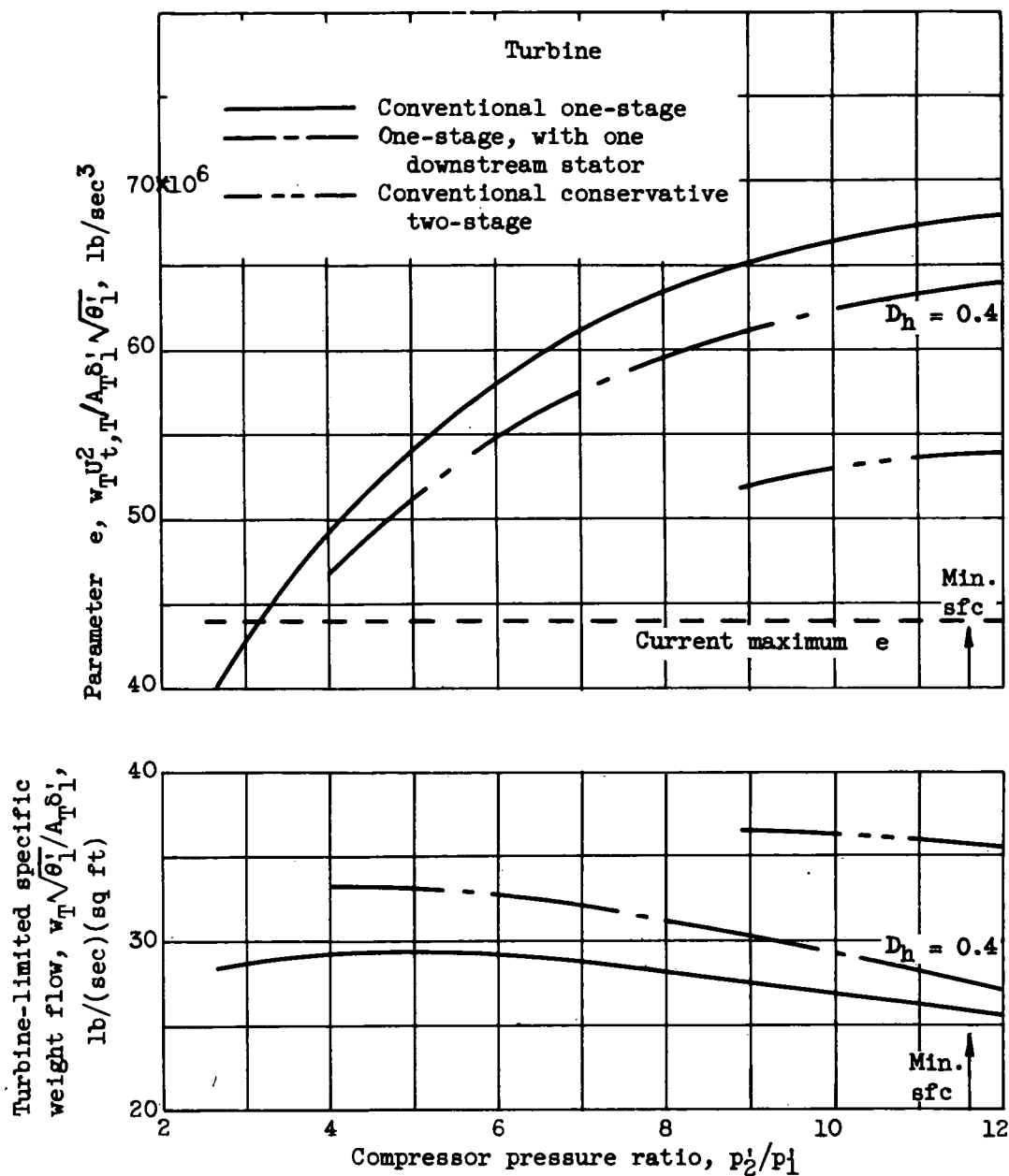






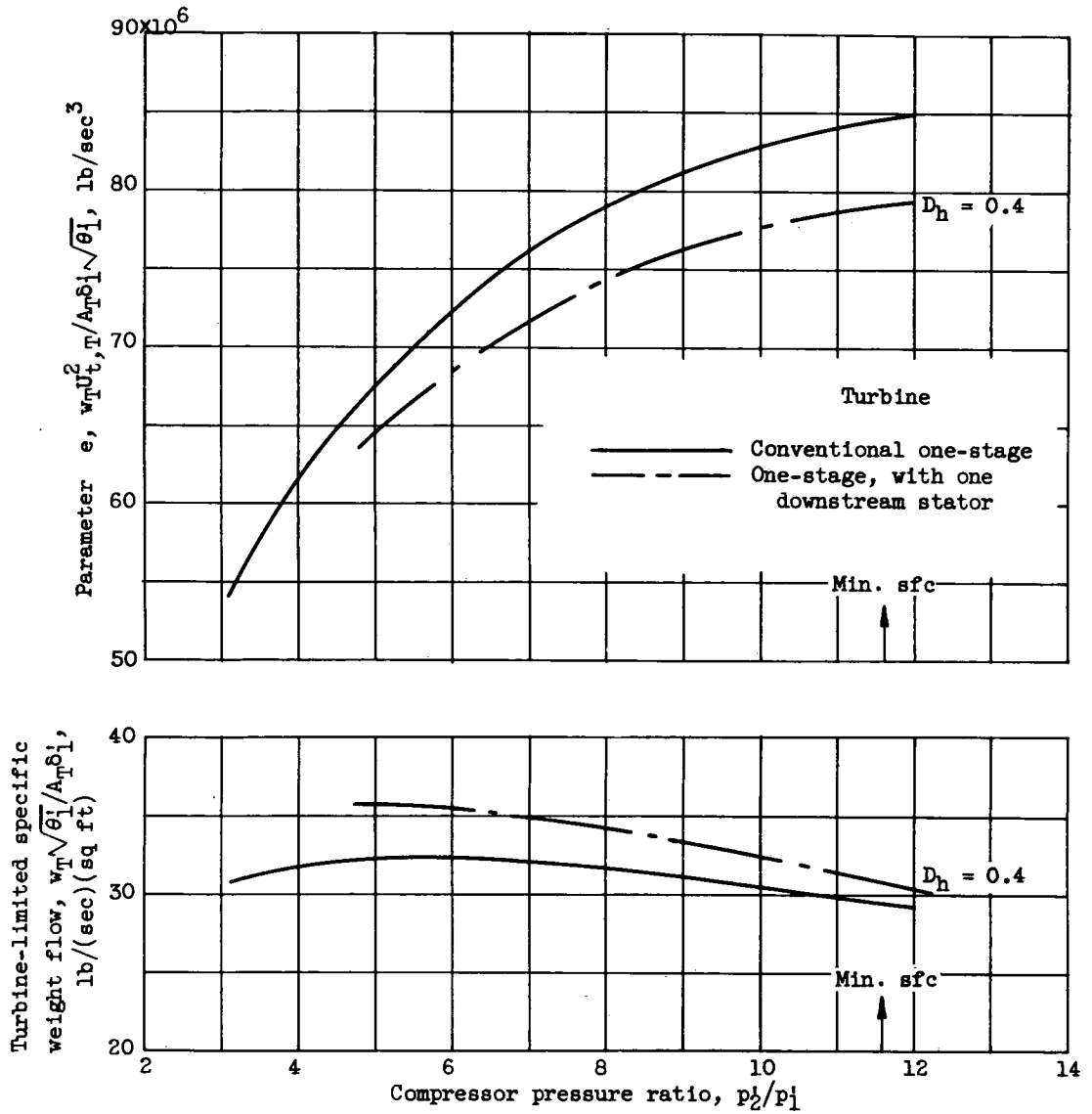
(b) Variation of turbine-limited specific weight flow and parameter  $e$  with compressor pressure ratio at turbine blade centrifugal stress of 30,000 psi.

Figure 2. - Continued. Turbine chart for sea-level static designs and for Mach 1.28 designs in stratosphere. Engine temperature ratio, 4.0.



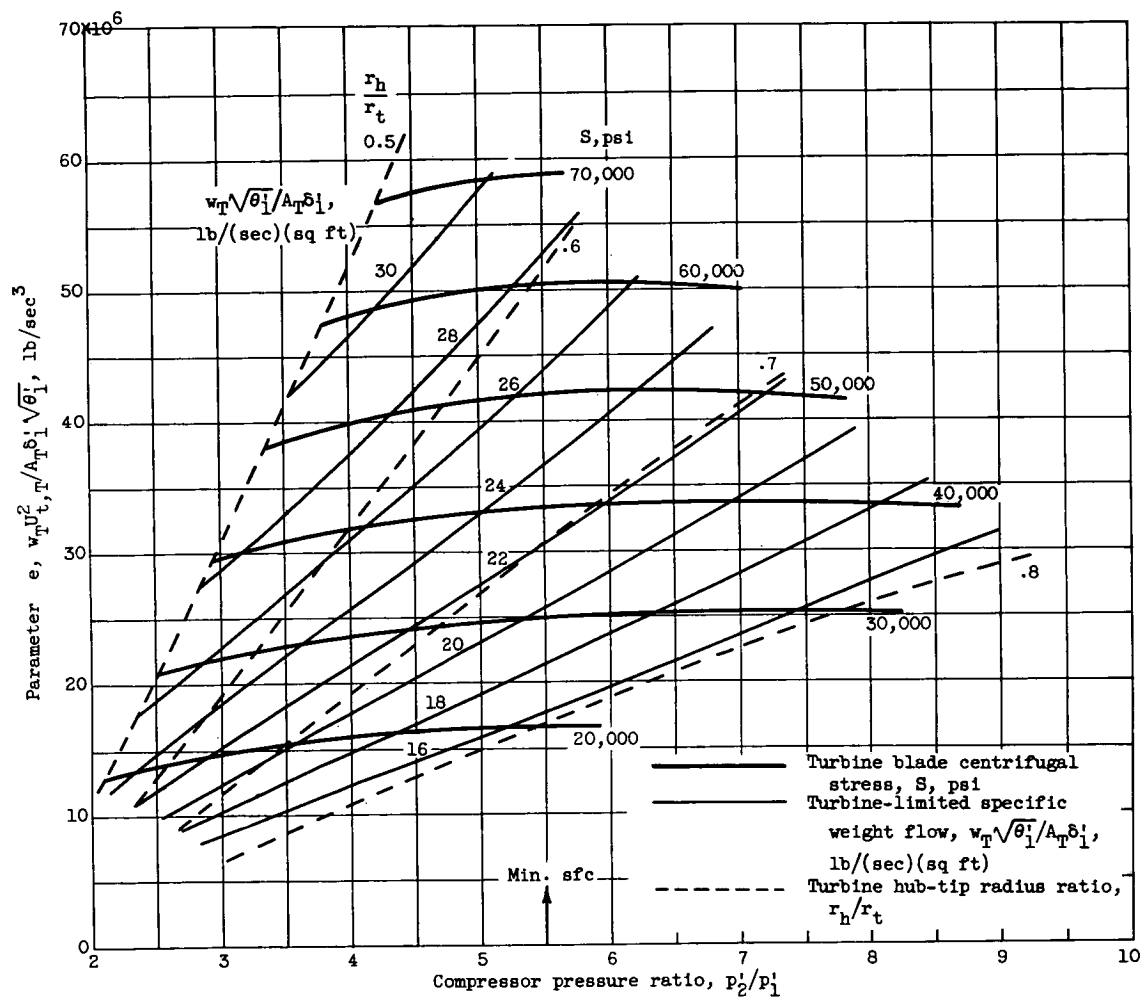
(c) Variation of turbine-limited specific weight flow and parameter  $e$  with compressor pressure ratio at turbine blade centrifugal stress of 40,000 psi.

Figure 2. - Continued. Turbine chart for sea-level static designs and for Mach 1.28 designs in stratosphere. Engine temperature ratio, 4.0.



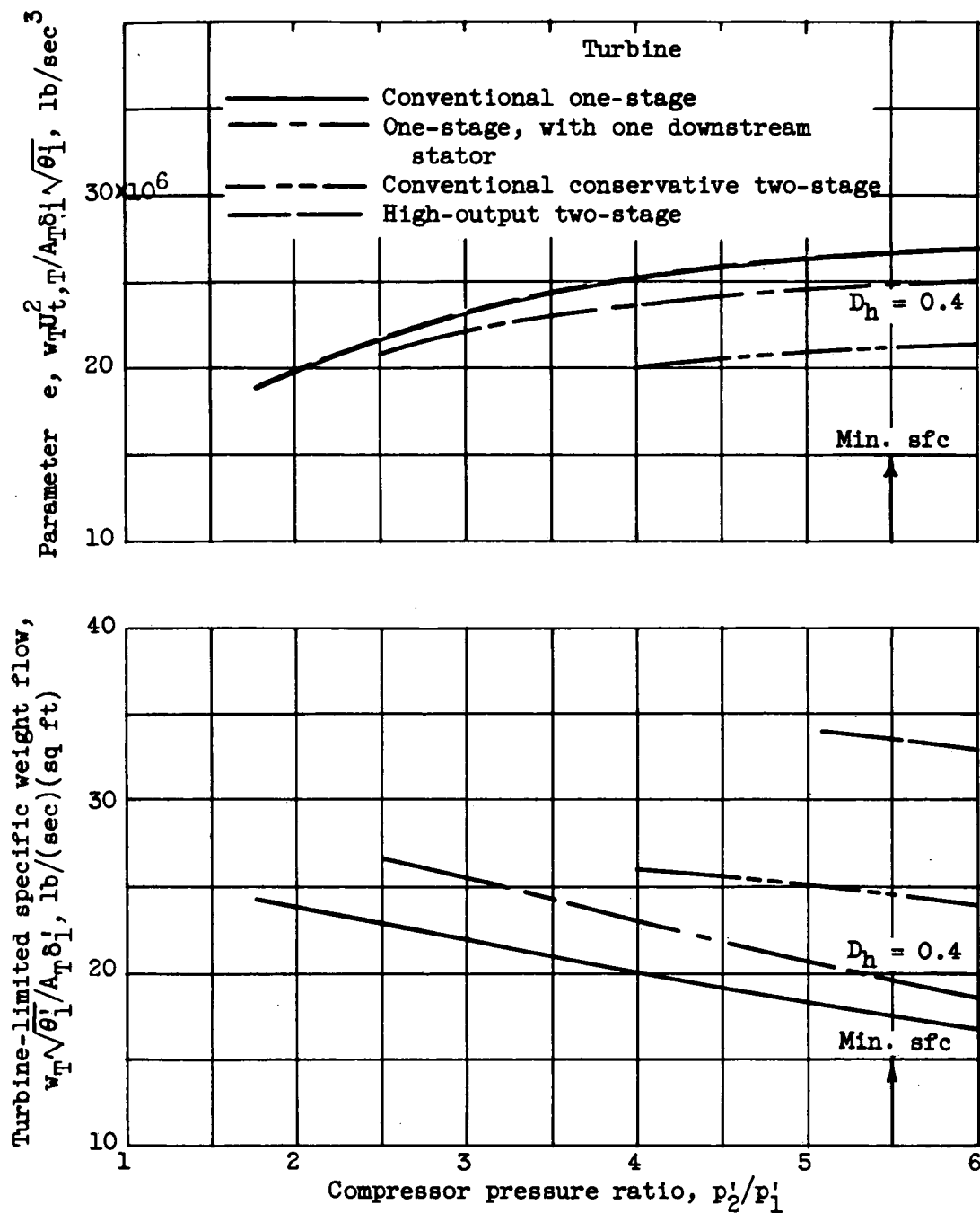
(d) Variation of turbine-limited specific weight flow and parameter  $e$  with compressor pressure ratio at turbine blade centrifugal stress of 50,000 psi.

Figure 2. - Concluded. Turbine chart for sea-level static designs and for Mach 1.28 designs in stratosphere. Engine temperature ratio, 4.0.



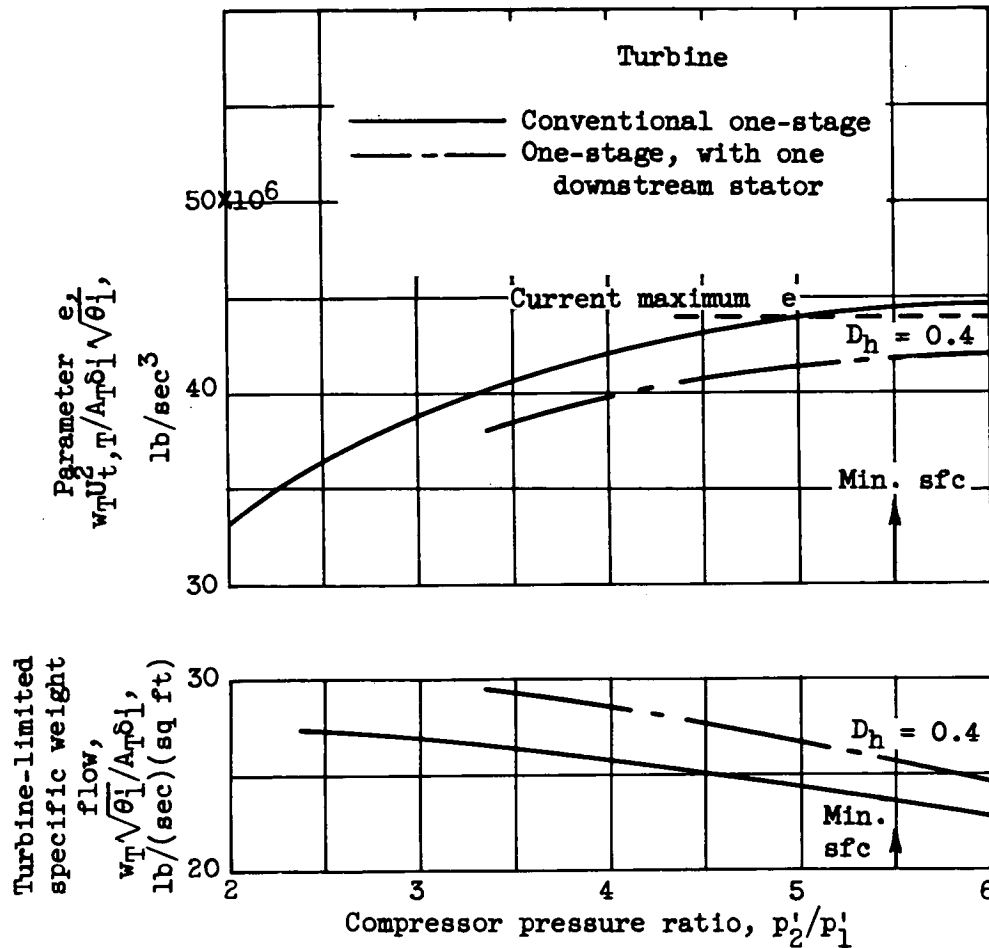
(a) Design characteristics for one-stage turbine with one row of downstream stator blades of 0.4 diffusion factor.

Figure 3. - Turbine chart for Mach 2.0 designs in stratosphere. Engine temperature ratio, 3.0.



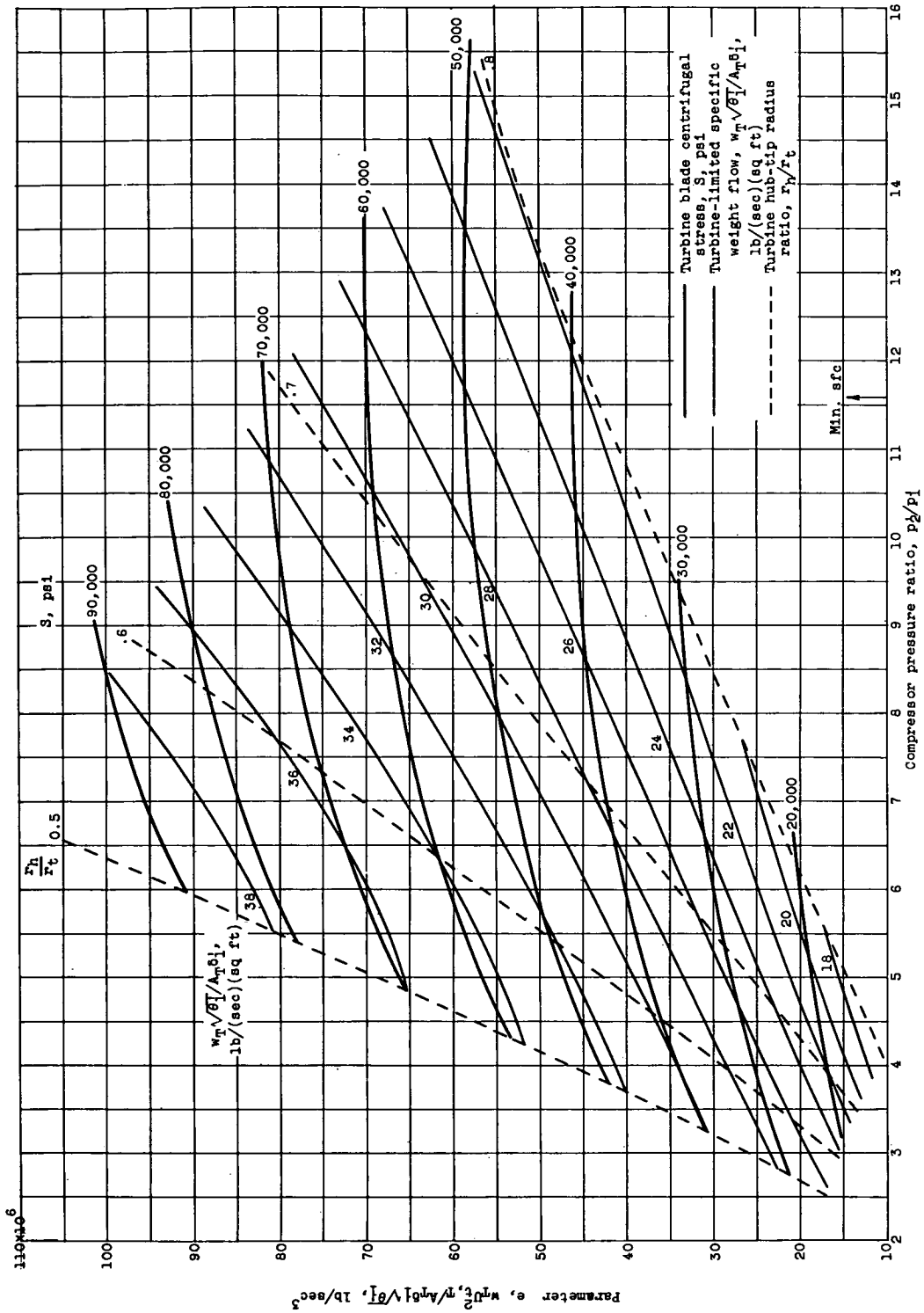
(b) Variation of turbine-limited specific weight flow and parameter  $e$  with compressor pressure ratio at turbine blade centrifugal stress of 30,000 psi.

Figure 3. - Continued. Turbine chart for Mach 2.0 designs in stratosphere. Engine temperature ratio, 3.0.



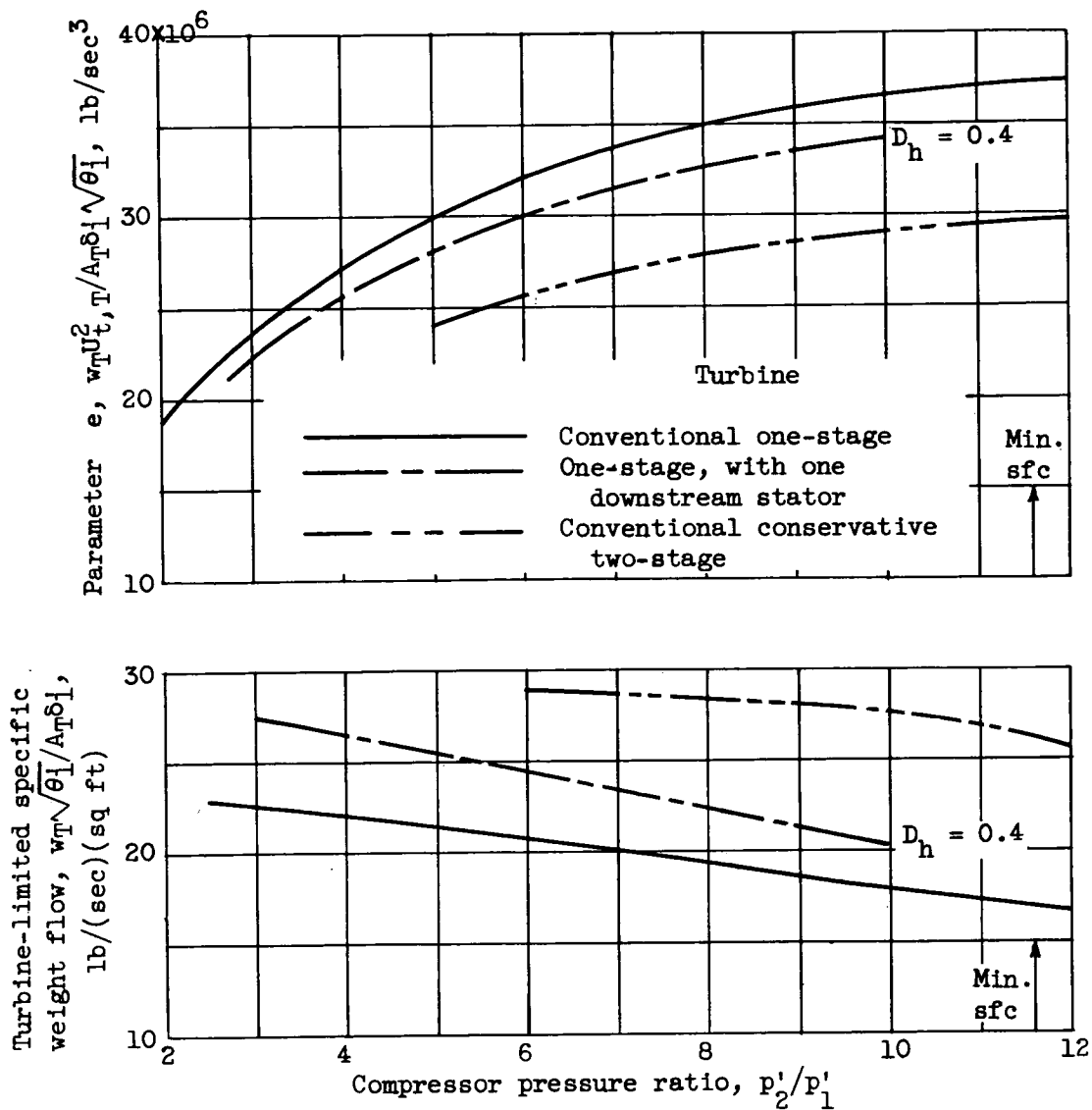
(c) Variation of turbine-limited specific weight flow and parameter  $e$  with compressor pressure ratio at turbine blade centrifugal stress of 50,000 psi.

Figure 3. - Concluded. Turbine chart for Mach 2.0 designs in stratosphere. Engine temperature ratio, 3.0.



(a) Design characteristics for one-stage turbine with one row of downstream stator blades with a diffusion factor of 0.4.

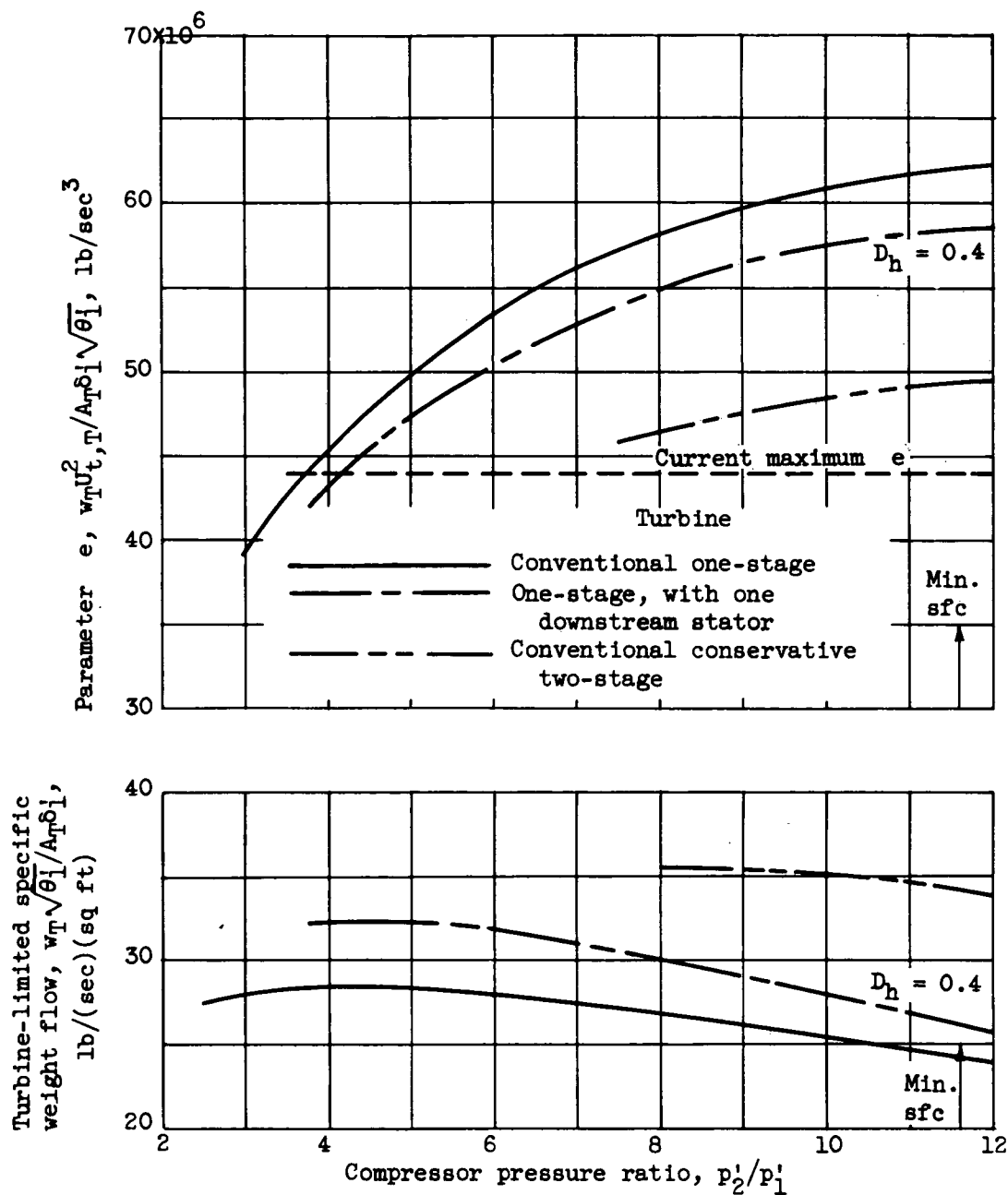
Figure 4. - Turbine chart for Mach 2.0 designs in stratosphere. Engine temperature ratio, 4.0.



(b) Variation of turbine-limited specific weight flow and parameter  $e$  with compressor pressure ratio at turbine blade centrifugal stress of 30,000 psi.

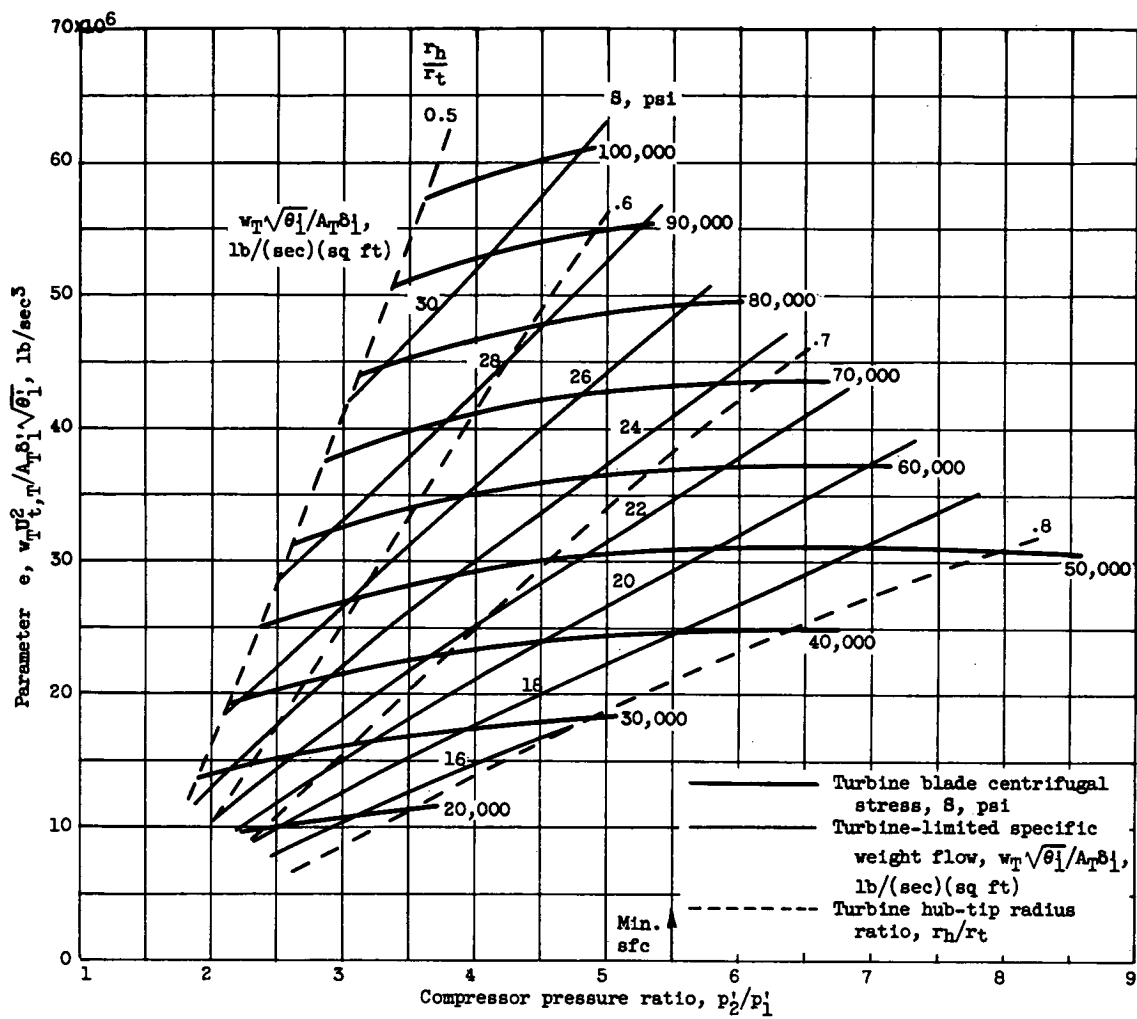
Figure 4. - Continued. Turbine chart for Mach 2.0 designs in stratosphere. Engine temperature ratio, 4.0.





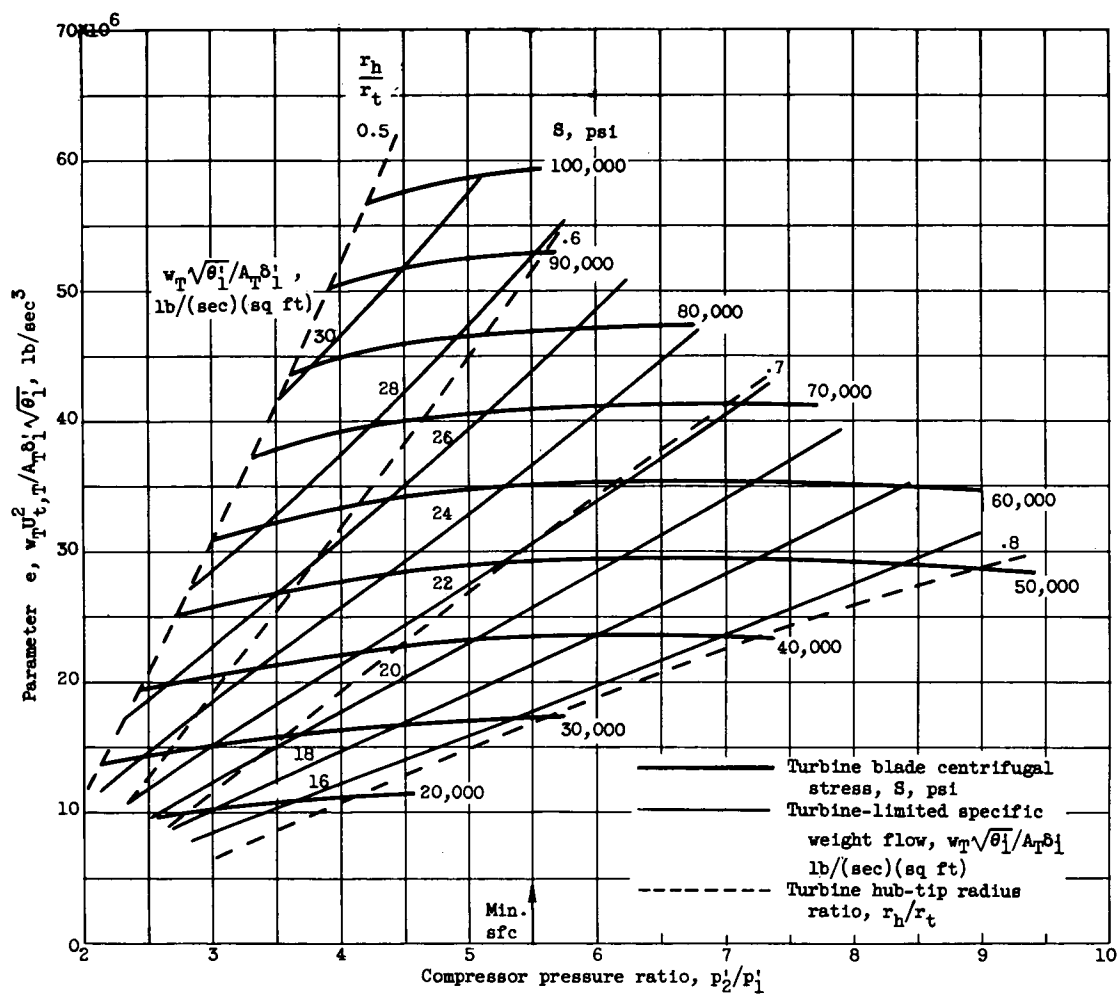
(c) Variation of turbine-limited specific weight flow and parameter e with compressor pressure ratio at turbine blade centrifugal stress of 50,000 psi.

Figure 4. - Concluded. Turbine chart for Mach 2.0 designs in stratosphere. Engine temperature ratio, 4.0.



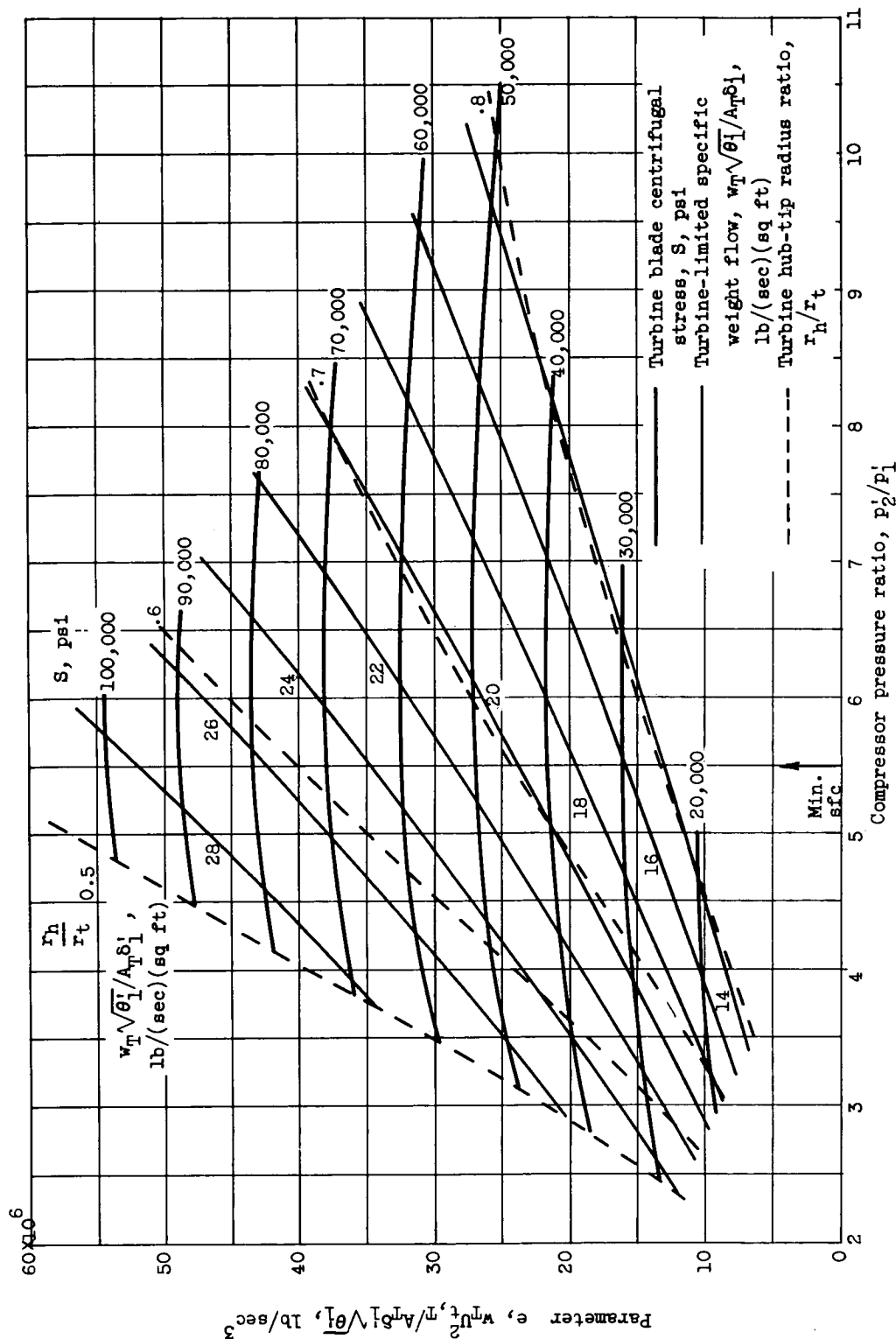
(a) Design characteristics for one-stage turbine with one row of downstream stator blades of 0.2 diffusion factor.

Figure 5. - Turbine chart for Mach 2.8 designs in stratosphere. Engine temperature ratio, 3.0.



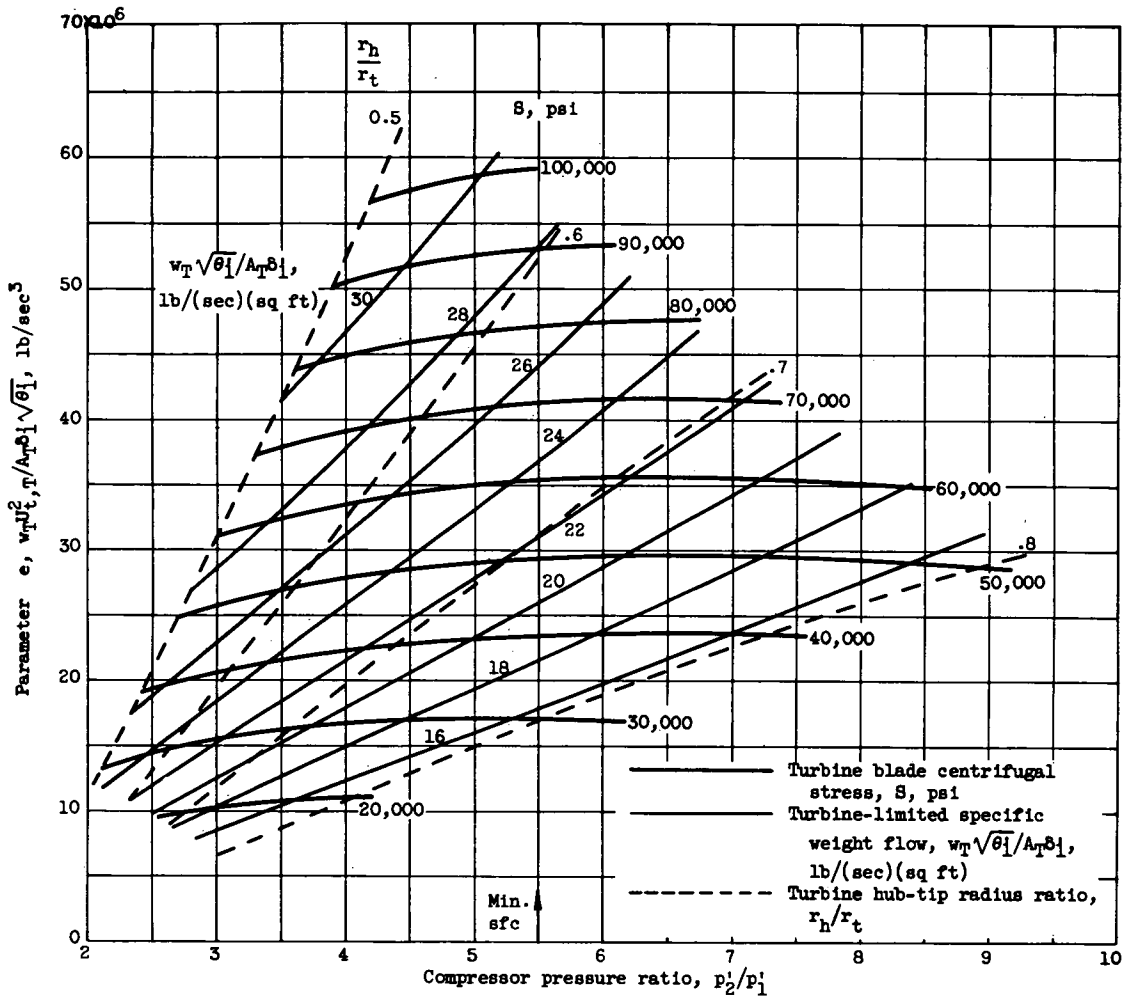
(b) Design characteristics for one-stage turbine with one row of downstream stator blades of 0.4 diffusion factor.

Figure 5. - Continued. Turbine chart for Mach 2.8 designs in stratosphere. Engine temperature ratio, 3.0.



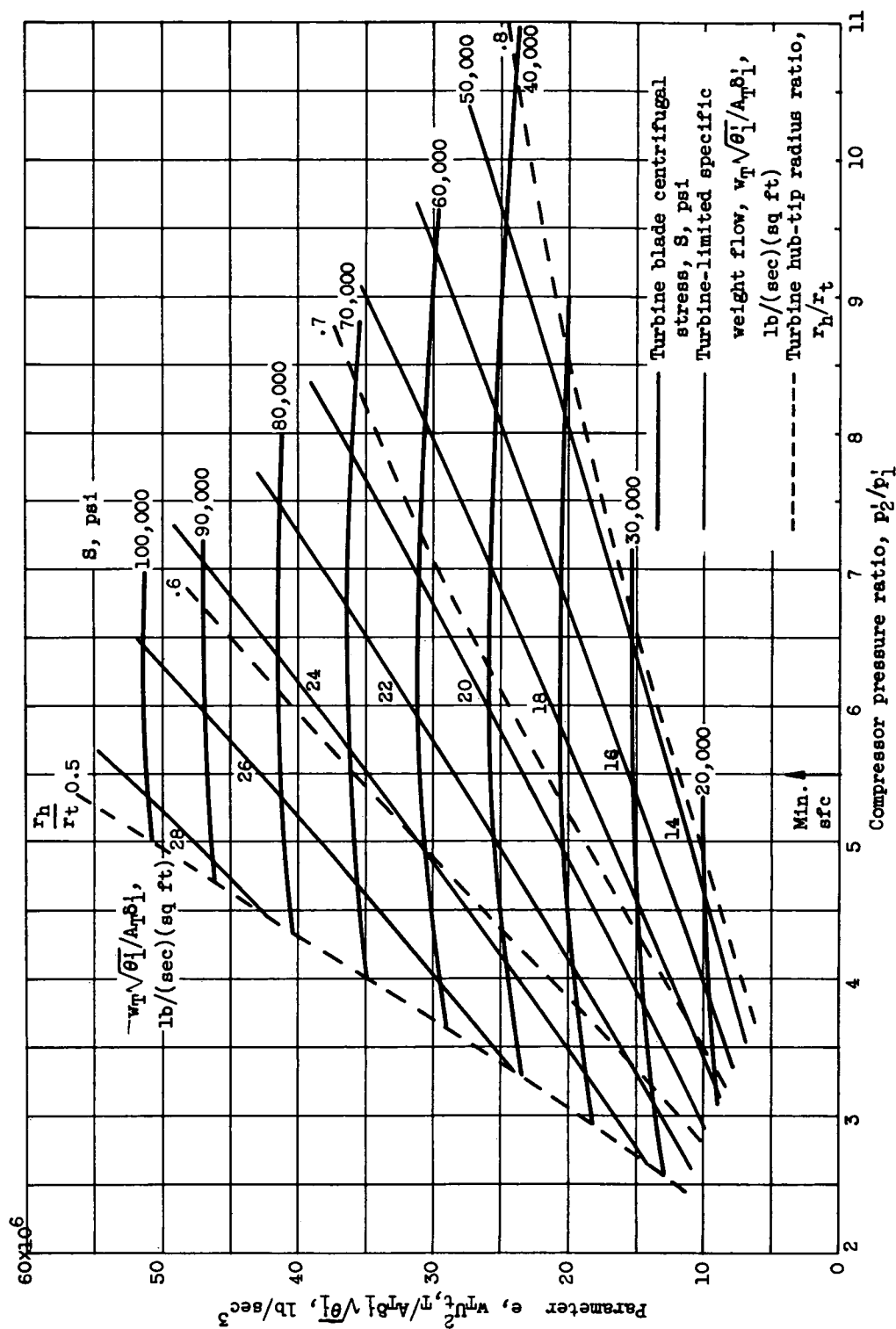
(c) Design characteristics for one-stage turbine with one row of downstream stator blades of 0.6 diffusion factor.

Figure 5. - Continued. Turbine chart for Mach 2.8 designs in stratosphere. Engine temperature ratio, 3.0.



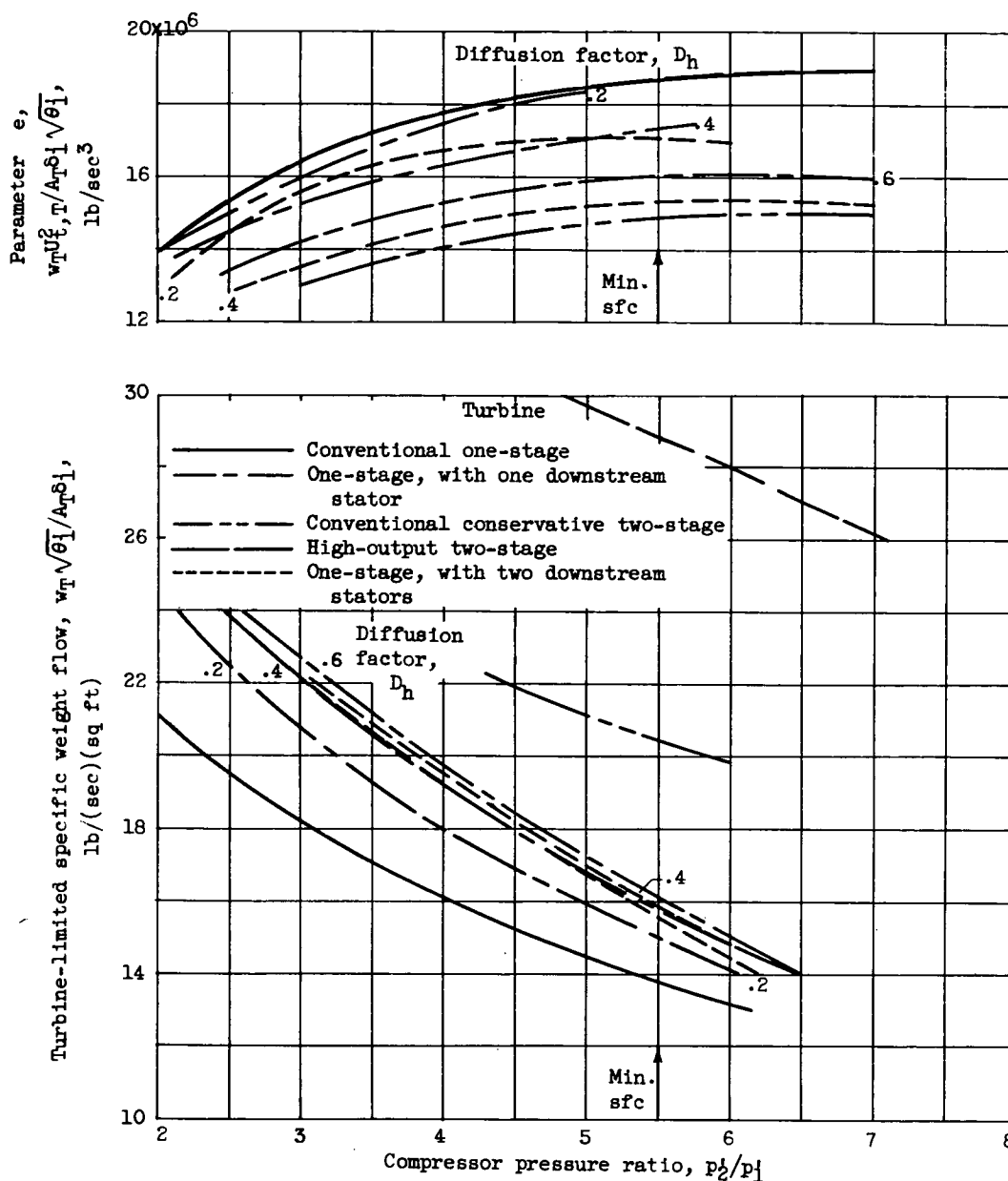
(d) Design characteristics for one-stage turbine with two rows of downstream stator blades of 0.2 diffusion factor.

Figure 5. - Continued. Turbine chart for Mach 2.8 designs in stratosphere. Engine temperature ratio, 3.0.



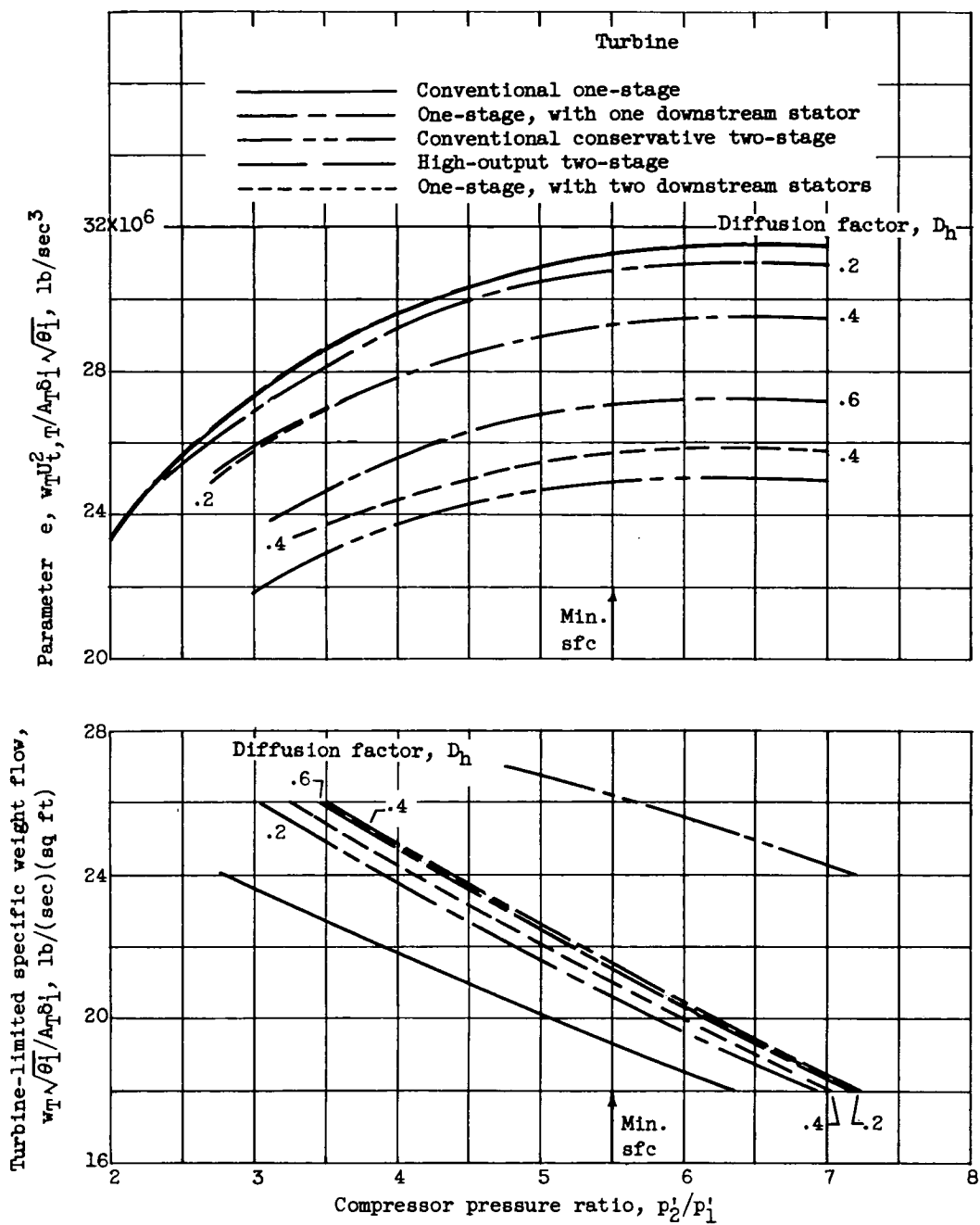
(e) Design characteristics for one-stage turbine with two rows of downstream stator blades of 0.4 diffusion factor.

Figure 5. - Continued. Turbine chart for Mach 2.8 designs in stratosphere. Engine temperature ratio, 3.0.



(f) Variation of turbine-limited specific weight flow and parameter e with compressor pressure ratio at turbine blade centrifugal stress of 30,000 psi.

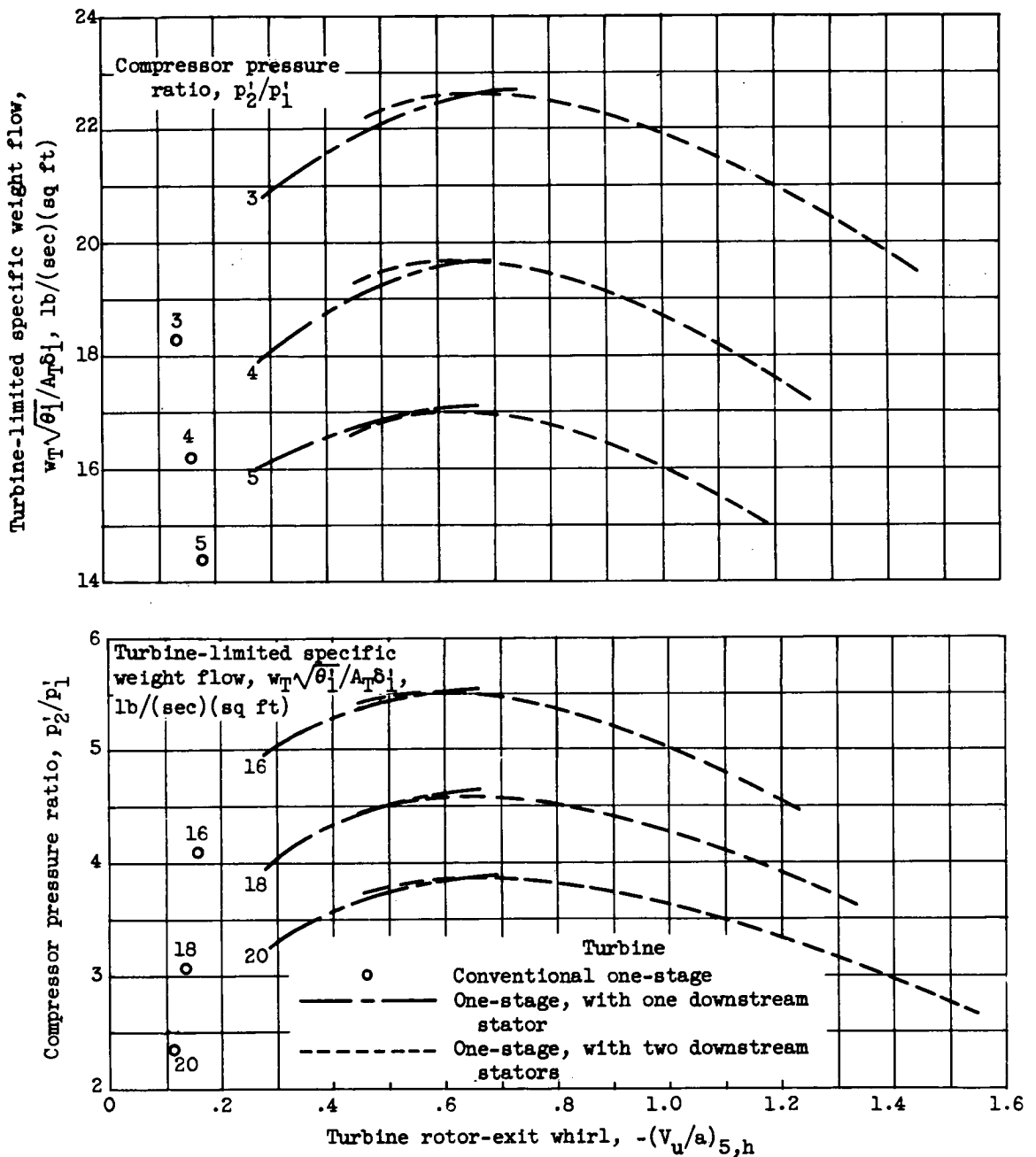
Figure 5. - Continued. Turbine chart for Mach 2.8 designs in stratosphere. Engine temperature ratio, 3.0.



(g) Variation of turbine-limited specific weight flow and parameter  $e$  with compressor pressure ratio at turbine blade centrifugal stress of 50,000 psi.

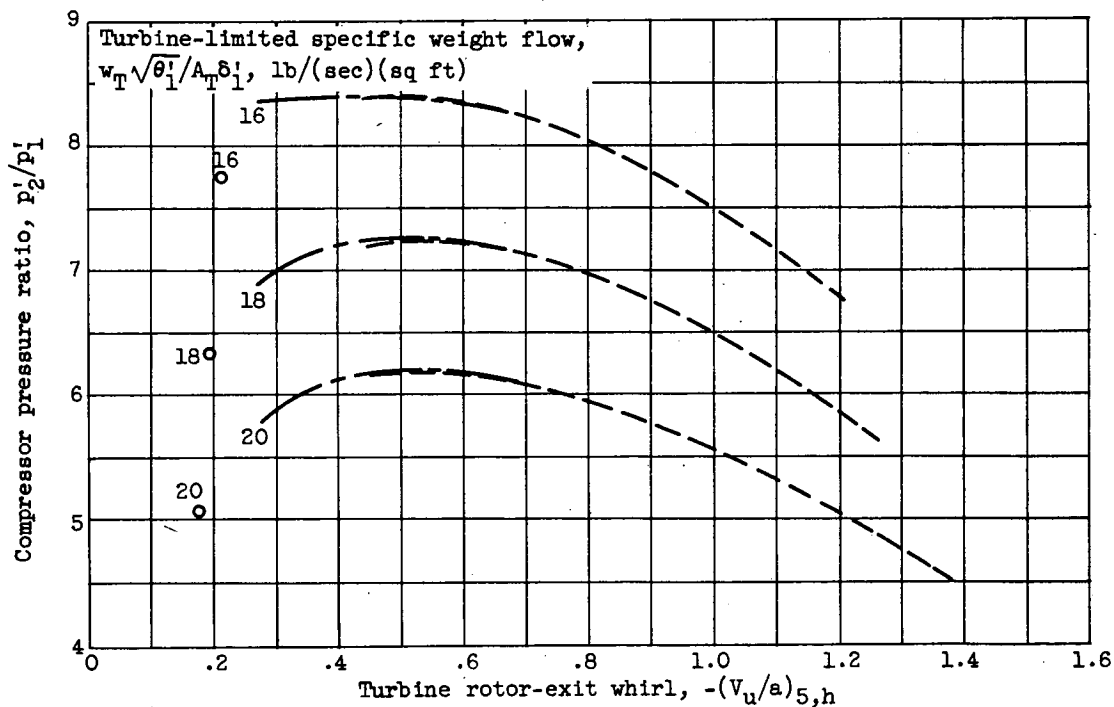
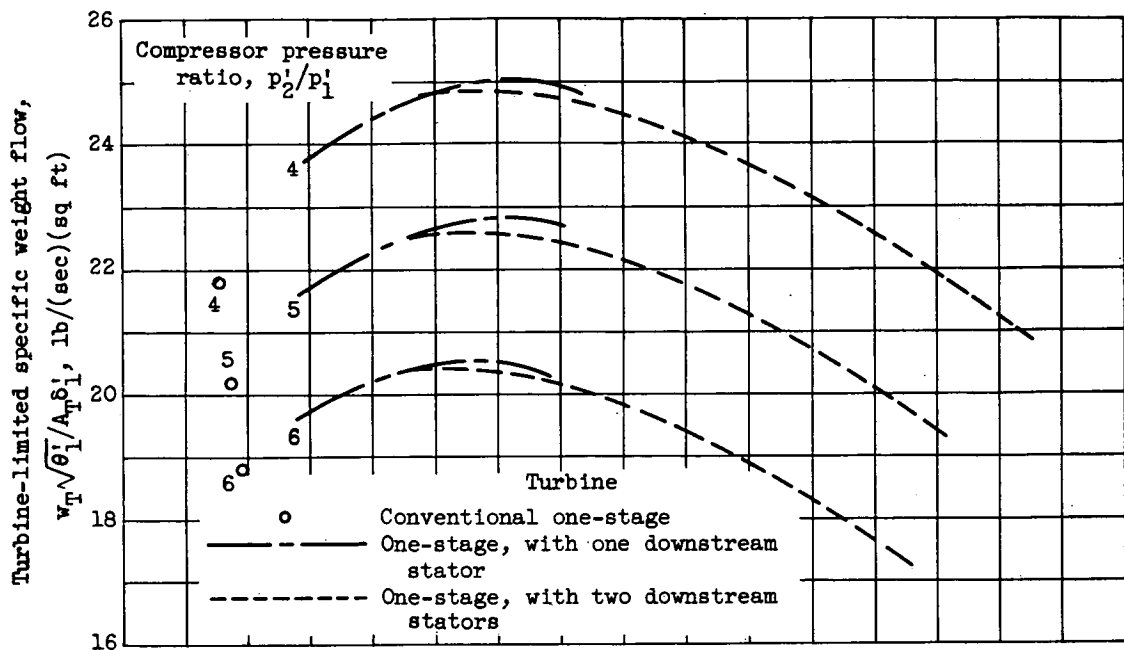
Figure 5. - Continued. Turbine chart for Mach 2.8 designs in stratosphere. Engine temperature ratio, 3.0.





(h) Variation of compressor pressure ratio and turbine-limited specific weight flow with turbine rotor-exit whirl at turbine blade centrifugal stress of 30,000 psi.

Figure 5. - Continued. Turbine chart for Mach 2.8 designs in stratosphere. Engine temperature ratio, 3.0.



(1) Variation of compressor pressure ratio and turbine-limited specific weight flow with turbine rotor-exit whirl at turbine blade centrifugal stress of 50,000 psi.

Figure 5. - Concluded. Turbine chart for Mach 2.8 designs in stratosphere. Engine temperature ratio, 3.0.